

Dominating set games
Bas van Velzen

e-mail: s.vanvelzen@uvt.nl
Tilburg University
Department of Econometrics and OR
THE NETHERLANDS

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ABSTRACT

We consider cooperative cost games arising from domination problems on graphs. A domination problem consists of a graph, a positive integer k , and a nonnegative function that assigns a fixed cost to each vertex. A k -dominating set is a subset of the vertices such that the distance between any vertex in the graph and at least one vertex in this subset is at most k . The k -domination problem is the problem of finding a so-called minimum weighted k -dominating set, i.e. a k -dominating set that minimizes the total cost of its vertices.

A motivation for domination problems is the following example. Consider a number of regions in which certain facilities are going to be placed. There is a fixed cost for the placement of a facility in a certain region. The problem is to select the regions in which to place facilities at minimum costs, such that each region is served by a facility in it or by a facility in a region nearby. The problem of placing the facilities at minimum costs can be regarded as a domination problem. Consider the graph where regions correspond to vertices and edges represent pairs of regions that are nearby. The fixed cost can be described by a function that assigns a nonnegative value to each vertex. The problem of placing the facilities at minimum cost is equivalent to finding a minimum weighted 1-dominating set.

A natural question that now arises is how to allocate the total costs of placing the facilities to the participating regions. We consider three cooperative cost games that model this cost allocation problem. The difference between these games lies in the possibilities that coalitions have of placing the facilities. In spite of the fact that the cost of coalitions may take different values in the three games, we derive one necessary and sufficient condition for the balancedness of all three games. Furthermore we obtain relations between the cores and we study concavity.