

## Long Run Equilibria in the Learning Processes

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### ABSTRACT

In this work we study behaviour dynamics of populations involved in a conflict (evolutionary games). The conflict is modelled by a finite game that is repeated in the long run. Members of the populations are myopic and make mistakes. We build two models that generalize, for any strategic finite game, the ideas of the paper "Learning, Mutation and Long Run Equilibria in Games" by Kandori, Mailath and Rob (KMR) [1993]. We prove, in both models, the existence of a Limit Distribution (LD) and of Long Run Equilibria (LRE) of learning processes. The obtained theorems are special cases of Friendlin and Wentzell theorems on perturbed Markov processes. We compare our work with the paper "Learning and Social Equilibrium in Large Populations" by D. Canning [1995]. In order to establish a comparison with Canning results, we adapt our second model, using dynamics where the players has memory of several previous periods. We prove there is always a LD and at least one LRE. If the LD is a unitary vector, the only one LRE corresponds to the Canning perfect social equilibrium. In the case of 2x2 symmetric games and a not unitary LD vector, for any population there are two LRE. When the population tends to infinity, both successions converges to the same limit, which corresponds to the perfect social equilibrium. In contrast with Canning theory, we do not need an infinite memory. It may remain relatively small.