

# A Characterization of Equitable Core Allocations in Cost-Sharing Games

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## ABSTRACT

A cost-sharing game can be defined over a finite set of players with transferable utilities who wish to cooperate in the provision of certain products or services. The centre of the game's imputation set, henceforth labelled CIS, is the unique cost allocation which distributes the total cost savings equally among the players. This allocation appears to be a natural choice of a cost-sharing rule on egalitarian grounds. If there are only two players, the CIS vector is called the standard game solution and constitutes the midpoint of the core of the game. In the general case of more than two players, however, the CIS vector does not necessarily occupy a 'central' core location. It may not even belong to the core. This weakness can be avoided by an egalitarian arbitration among coalitions which minimizes dissatisfaction in making the least satisfied coalitions as well off as possible. The resulting cost allocation is called the pre-mailnucleolus (point) of the game and is always an element of a nonempty core. A drawback is that the problem of calculating the pre-mailnucleolus of a general cost-sharing game does not have a polynomial complexity, i.e., the associated minimal order of computational resources which are required to solve this problem is not a polynomial in the number of players. Therefore, it would be convenient to know a priori whether the pre-mailnucleolus of a particular game coincides with the outcome of a cost-allocation method which is computationally less demanding. For example, the popular separable-mailcosts remaining-benefits (SCRB) approach to cost allocation in the development of water resources has been observed to yield the pre-mailnucleolus of certain cost-sharing games. In this paper, we study classes of games for which the pre-mailnucleolus coincides with the CIS vector of cost shares. The paper is organized as follows: Section 2 presents our notation as well as formal definitions of the basic solution concepts. In Section 3, we establish with respect to coalitional excess and cost savings several conditions which are either necessary or sufficient for a coincidence of the pre-mailnucleolus of a cost-sharing game and the game's CIS vector. We find that a coincidence is more likely when the cost savings of the grand coalition of all players are large in comparison to the stand-alone cost savings of all other coalitions, e.g., as the cost-savings function is strongly convex.