

Shapley Value in a Differential TU-game with Nonlinear Payoff Function

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ABSTRACT

In this paper we investigate a n -person nonzero-sum pursuit game. We construct both a game in normal form and its TU-cooperative version, and find their solutions (the core, the Shapley value, the Nucleous). We studied the problem of nonemptiness of the core for the considered model of pursuit and its time-mailconsistency. We proved that the core in this particular n -person game is never empty and, moreover, is always time-mailconsistent.

Studying this problem we found out an unusual and interesting fact, which is going to be the main object of the present work. Namely, that is a very unexpected behaviour of the Shapley value in the considered game. It is certainly true that the Shapley value is by far the most important and widely used fair division method in static TU-games, however, in dynamic TU-games the situation changes completely. Our aim is to demonstrate this fact on an example. It turns out that depending on positions of the players this cooperative differential game can happen to be convex or not. Then, moving along the cooperative trajectory the players in some sense travel over the subgames, which can turn out to be either convex or not. And moreover, when the game is proceeding we can watch the process of changing it from one form into another.

In other word, being convex at the initial moment the game can change into a non-convex one, and backwards. We prove that the Shapley value is inconsistent here. During the game it is rushing about the core, which is never empty in this game. Namely, it falls into the core and then in some time goes out of the core. So, it is not worth choosing the Shapley value as an allocation in the game. The only conclusion here can be that the Shapley value is unreasonable to be taken as a solution in some dynamic TU-games. It can behave like in illustrative examples which are going to be given in the full paper.