Duopoly with Piecewise Linear Discontinuous Reaction Functions

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ABSTRACT

We consider a variant of Palander's model of duopoly with a piecewise linear (continuous) *demand function* of the form

$$p = f(x+y) = \begin{cases} \alpha_1 - \beta_1(x+y), & 0 \le x+y \le q; \\ \alpha_2 - \beta_2(x+y), & q < x+y \le q_1; \\ 0, & x+y > q_1; \end{cases}$$

where $q = (\alpha_1 - \alpha_2)/(\beta_1 - \beta_2)$ and $q_1 = \alpha_2/\beta_2$ are kink points; $\alpha_i > 0, \beta_i > 0, \ i = 1,2$.

The *cost function* of the competitors are given by

$$C_x(x) = a_x x - b_x x^2, \qquad 0 < x < \frac{a_x}{2b_x};$$
$$C_y(y) = a_y y - b_y y^2, \qquad 0 < y < \frac{a_y}{2b_y}.$$

We propose an (economic) example which generalizes the model studied by Palander (1936), and show that in spite of the narrow intervals in which parameters β_1 and β_2 are varied, the model reveals a very rich dynamic behavior.

As in the general case of duopoly, the dynamics of our model can be described by means of the properties of the piecewise linear one-dimensional maps $F : \mathbb{R}^+ \to \mathbb{R}^+$ and $G : \mathbb{R}^+ \to \mathbb{R}^+$ given by $F : x \mapsto F(x) = (\varphi \circ \psi)(x)$ and $G : y \mapsto G(y) = (\psi \circ \varphi)(y)$, where $\varphi(y)$ and $\psi(x)$ are the corresponding *reaction functions* of the two competitors, which appear to be *discontinuous piecewise linear* functions. Depending on the parameters the map F (and G) may be made up by different numbers of linear pieces, up to 5.

A two-dimensional bifurcation diagram in the (β_1, β_2)-parameter plane is presented, where regions of existence of attracting cycles of different periods are indicated. We also give conditions for the existence of a chaotic trajectory for an open set in the parameter plane.

References

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