The complete solution of the problem of kind in the linear differential games of pursuit and evasion

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ABSTRACT

The problem of kind in differential games of pursuit and evasion was first formulated by Rufus Isaacs [2]. Since then, this problem has been studied extensively by many authors in various investigations [3], [4].

Let us consider a linear differential game of pursuit and evasion described by the controlled system: where

$$\dot{z} = Cz - u + v,$$

$z \in R^d$, C-constant $d \times d$ matrix, u,v- control parameters of pursuit and evasion and $u \in P$, $v \in Q$. Control domains $P,Q \subset R^d$ are assumed nonempty convex compacts. The terminal domain M is a nonempty open subset of R^d. Let us assume that $X=R^d \backslash M$.

The pursuer and evader use strategies defined by A.Azamov [1] as lower and upper strategies respectively.

Definition. $U=(\varepsilon, u_\varepsilon, t_\varepsilon)$ is the pursuer's lower strategy where ε is a positive constant,

$u_\varepsilon: R^+ \times R^d \to P([0; \varepsilon])$, $t_\varepsilon: R^+ \times R^d \times Q([0;\varepsilon]) \to (0; \varepsilon]$, $R^+=[0;\infty)$.

Definition. The evader's upper strategy is called the family of operators $\{v^\varepsilon\}$, indexed by the positive parameter ε :

$v^\varepsilon: R^+ \times R^d \times P([0;\varepsilon]) \to Q([0;\varepsilon])$.

The set of all lower strategies of the pursuer we denote by P_*, and all evader's upper strategies by Q^*.

Definition. We shall say that from an initial point $z_0 \notin M$ it is possible to complete the pursuit, if there exists a strategy $U \in P_*$, such that for any strategy $V \in Q^*$ the following is fulfilled:

$z(t,z_0,U,V) \in M$ for some $t \geq 0$.

Definition. We shall say that from an initial point $z_0 \in X$ it is possible to evade, if there exists a strategy $V \in Q^*$ such that for any strategy $U \in P_*$ the following inclusion is fulfilled:

$z(t,z_0,U,V) \in X$ for any $t \geq 0$.

The problem of kind is to find the sets of initial points X^+ from which it is possible to complete the pursuit and $X^-=X \backslash X^+$ from which it is possible to evade.

For given differential game of pursuit and evasion with the class of strategies defined by A.Azamov the problem of kind is completely solved.

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