

Iterated weak dominance and subgame dominance

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ABSTRACT

For the class of normal form games where a finite number of players have strict preferences over a finite set of outcomes (utility vectors), Gretlein (Dominance Elimination Procedures on Finite Alternative Games, *Int J Game Theory* 12, 107-113, 1983) showed that the set of outcomes resulting from iterative elimination of some weakly dominated strategies contains the set of outcomes that remains from a procedure that removes all weakly dominated strategies in every step until no more strategies can be removed. In this paper, we extend this result and show that any set of strategies surviving k rounds of elimination of some weakly dominated strategies can be reduced to a set of strategies equivalent with the strategies surviving k rounds of elimination of all weakly dominated strategies in every round. The reduction can be carried out by (at most k) further rounds of elimination of weakly dominated strategies (Theorem 1). From this we obtain a strengthened version of Gretlein's result (Corollary 1). Moreover, the result extends to a class of games satisfying transference of decisionmaker indifference which is less restrictive than strict preferences over outcomes.

We then consider applications and demonstrate how we may obtain a unified approach to the above-mentioned result by Gretlein and recent work by Ewerhart (Iterated Weak Dominance in Strictly Competitive Games of Perfect Information, *J Econ Theory* 107, 474-482, 2002) and by Marx & Swinkels (Order Independence for Iterated Weak Dominance, *Games Econ Behav* 18, 219-245, 1997).

Our first application is to finite, two-player strictly competitive games of perfect information. We consider iterative elimination of weakly subgame dominated strategies, a procedure that intuitively can be viewed as the removal of certain 'dominated' branches from the game tree in every step. We show that any game with at most n outcomes is dominance solvable by $n-1$ rounds of elimination of all weakly subgame dominated strategies (Theorem 2), and give a short and elementary proof. By combining the results (Theorem 1 and 2), we also obtain the result by Ewerhart who show that any game with at most n outcomes is dominance solvable by $n-1$ rounds of elimination of all weakly dominated strategies (Corollary 2).

Our second application is to work by Marx & Swinkels. For the class of normal form games satisfying transference of decisionmaker indifference, Marx & Swinkels show order independence: Regardless of the order in which weakly dominated strategies are removed, any two full reductions are the same up to removal of redundant strategies and renaming of strategies. We round off by formulating this as a corollary of our main result (Corollary 3).