

## Counting power indices for games with a priori unions

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### ABSTRACT

We propose a way of extending "counting" power indices to simple games with a priori unions (precoalitions), related to Owen's [1] construction of Shapley value - and, subsequently, of any value for cooperative games - for games with a priori unions. Counting indices are those power indices which are computed by summing appropriate coefficients - which, for a given coalition, depend only on the values of the characteristic function of the game on its subsets, and possibly on its size - over all coalitions in which the relevant player is decisive, and then by normalization. This class contains, among others, the Shapley value and Banzhaf, Johnston, Holler and Deegan - Packel indices.

The construction is based on the observation that any given coalition in the "external" game (i.e., any set of precoalitions) defines on each of the remaining precoalitions an "internal" game which is a simple game. This enables extending counting indices to games with a priori unions without necessity of "generalizing" them to all TU games. The indices obtained in this manner are consistent in the sense that individual power indices of players in a precoalition sum up to the index of that precoalition as a player in the external game. Our extension of the Shapley value coincides with that of Owen, but for other indices obtained by normalizing weighted sums of marginal contributions vectors the two extensions generally lead to different indices.

[1] G. Owen, Values of games with a priori unions, in : Essays in mathematical economics and game theory, R. Henn and O. Moeschlin eds., Springer - Verlag 1977