

## Time Consistency and Time Discounting

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### ABSTRACT

In the economic literature the most widely used type of additive time discounting is Exponential Discounting (ED). Recent work however casts doubts on its ability to explain how individuals effectively choose. In particular a more general form of discounting that gained importance, in both applied and theoretical work, is the Hyperbolic Discounting model, which well captures phenomena such as procrastination and addiction. Such phenomena are manifestations of time inconsistent choices which can not be explained by ED, the only additive discounting model which guarantees time consistency. Another way to look at the association between time consistency and ED is the following. Let  $t=0,1,2,\dots$  be the time index and consider a decision maker evaluating, at  $t=0$ , an infinite stream of unitary payoffs available at each  $t=0,1,2,\dots$ . Moreover, let  $0 < d < 1$  be his discount rate and  $W(t)$  his welfare (generated by the payoffs stream) at time  $t$ . Then  $W(0) = 1 + d + d^2 + \dots + d^t + \dots = 1 + d + d^2 + \dots + d^t W(t) = 1 + d + d^2 + \dots + d^t W(0)$  so that  $W(t) = 1/(1-d)$  for all  $t$ . Namely, in such a stationary environment (given by the stream) the decision maker perceives the same welfare at each  $t$ ; we interpret this as a manifestation of time consistency on the part of the decision maker. The expression for  $W(0)$  calculated in terms of  $W(t)$  can be interpreted as "self 0" (in evaluating his welfare) being egoistic up to time  $t-1$  and then altruistic towards one self, namely "self  $t$ ". It is then natural to ask what kind of discounting function characterises a dynamically consistent (in the above sense) decision maker, egoistic up to time  $t-1$  and then altruistic towards  $k > 1$  selves. Letting now  $d(0)=1$  and  $d(t)$ , with  $t=1,2,\dots$ , be the generic discounting function, the relevant functional would be  $W(0) = 1 + d(1) + d(2) + \dots + d(t-1) + d(t)W(t) + \dots + d(t+k)W(t+k)$ . Time consistency implies  $W(t)=W$  for all  $t$ . Then, the main result of the paper shows that  $d(i)$  takes the form of a generalised ED and that, in particular, can be very close to the Quasi-Hyperbolic Discounting  $1, b d, b^2 d^2, \dots, b^t d^t, \dots$ , introduced by Phelps-Pollack (RES;1968) and more recently analysed by other scholars such as Laibson (QJE, 1987). Hence, the analysis could characterise, in terms of the above notion of time consistency, discounting models other than ED when agents are altruistic towards more than one future self.