

Alpha-decisiveness in simple games

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KeyWords:multilinear extension, a priori unions and Banzhaf value theories

ABSTRACT

In a recent paper (Carreras [2001]), and based on a neutral probabilistic voting model for a proposal P against a given status quo Q, a decisiveness index has been introduced that applies to any simple game and measures the structural agility of the game as a system to take decisions. The index takes values between 0 and 1 and, e.g., all decisive (i.e. proper and strong) games get $\frac{1}{2}$. Main properties of the index with regard to composition of games, several axiomatic characterizations, and a close potential-like relationship to the Banzhaf value (Owen [1975]) that gives rise to a computational procedure, are stated in the above-mentioned paper.

The decisiveness index of a game can be calculated in terms of the multilinear extension (MLE) of the game (Owen [1972]) by replacing each variable with $\frac{1}{2}$. This suggests that, in fact, any values of the variables might make sense, and it leads to a new, generalized voting model and its corresponding decisiveness and Banzhaf indices.

Thus, by assuming an assessment a is given that reflects the inclination of each player to vote for P, we define a new a -decisiveness index and check that it can be computed by means of the MLE of the simple game to which it applies. Moreover, the analogue of the relationship between decisiveness and the Banzhaf value gives rise to a definition of a Banzhaf a -value, and its computation in terms of MLE is also provided, always generalizing the formal case.

The new probabilistic model allows us to analyze strategic voting aspects that the formal model cannot cope with, as e.g. the differences on decisiveness between two decisive games that should appear when one of them has many players and the other has few players, or the effects of imposing voting discipline within parties on the decisiveness of a parliamentary body. Other applications of the decisiveness notion can be derived that relate to restricted communication and cooperation, by means of, say, affinities, incompatibilities or both, or coalition structures and quotient games; in this latter case, Owen's modification of the Banzhaf value (Owen [1982]) plays, of course, an important role. Some examples will illustrate the above ideas.

References

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