

Extending Owen's union concept: Superplayers, their agreements, and the resultant solution for cooperative games.

Mike Ball

e-mail: ballm@liv.ac.uk
University of Liverpool
Mathematical Sciences
U.K.

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ABSTRACT

A Superplayer is a set of players who agree (0) to act as if they were one player cf. the dictionary definition of a coalition. Superplayers maximise their payoff, and, in superadditive games, join with other superplayers to form a superplayer whose player-set is the union of the component sets. Thus a superplayer's structure is that of a rooted sub-tree, each edge representing a set and twigs the individual players. An edge at the root represents the superplayer's set. A grand superplayer, with set N , has a tree-structure; cf. Owen's union concept. Instead of a linearity axiom and a value, we find the optimal c -trees, i.e. a tree and an associated imputation.

A move by a superplayer means a change of tree. A transition is between two c -trees and is a move and a change of imputation. The players of a superplayer agree:

- (i) to keep secret the structure of the superplayer:
- (ii) that no disjoint superplayer may join them if the payoff to any component player on that move and any immediate extra move by the joining superplayer decreases a player's payoff.

(i), (ii) and (0) define a superplayer.

Superplayers calculate the transitions, and opt for a play in the solution. Agreement (i) means that if two superplayers, with the same player-set but different structure, occupy the same position, then in the solution they get the same payoff. Agreement (ii) means that transitions result in the solution set only containing c -trees with a full-binary structure.

The payoffs to the superplayers at the tree-root are determined by allocation parameters. Given these, transitions are operations between c -trees. The operator puts the c -trees into partially-ordered equivalence classes, called tree-classes. Reversible transitions also classify c -trees into equivalence classes contained within the tree-classes. The solution set is the equivalence class, a standard equilibrium, in the extreme tree-class where any move by a superplayer does not alter the payoff of that superplayer. When all full-binary trees occur in the solution, there is one (calculable) c -tree to each tree. Certain allocation parameters give results whose average is the Shapley value. An example is given, if time permits.