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"THE EFFECTS OF A GREEN MONETARY POLICY ON FIRMS FINANCING COST"

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The effects of a green monetary policy on firms financing cost

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Abstract

The monetary policy operations of a Central Bank (CB) involve allocation decisions when purchasing assets and taking collateral. A green monetary policy aims to steer or tilt the allocation of assets and collateral towards low-carbon industries, to reduce the cost of capital for these sectors in comparison to high-carbon ones. Starting from a corporate bonds purchase program (e.g. CSPP) that follows a carbon-neutral monetary policy, we analyze how a shift in the CB portfolio allocation towards bonds issued by low-carbon companies can favor green firms in the market. Relying on optimal portfolio theory, we study how the CB might include the risk related to the environmental sustainability of firms in its balance sheet. In addition, we analyze the interactions between the neutral or green CB re-balancing policy and the evolutionary choice (i.e. by means of exponential replicator dynamics) of a population of firms that can decide to be green or not according to bonds borrowing cost.

Keywords: Monetary Policy; Optimal Portfolio Allocation; Environmental Economics; Interacting Agents; Evolutionary Dynamics.

JEL codes: E52, E58, G11, C61, C73, Q50.

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1 Introduction

The core operations of a Central Bank (CB) include conducting monetary policy operations, managing foreign exchange reserves, and operating large value payment systems. These core operations, for which we use the shorthand of monetary policy operations, involve allocation decisions when purchasing assets and taking collateral, through the so-called 'eligibility criteria'.
The major CBs accept private sector papers (corporate bonds, bank bonds, and bank loans) for asset purchases and collateral, and this credit policy practice has been further intensified under quantitative easing after the global financial crisis. As for the European Central Bank (ECB), the largest items on the Eurosystem balance sheet are securities holdings under the Asset Purchases Program (APP), which was launched in October 2014, and loans to EU credit institutions as part of monetary policy operations. Since then, several Asset Purchase Programs (APPs) have been introduced, allowing the ECB to buy government bonds (PSPP), asset-backed securities (ABSPP) and covered bonds (CBPP3). On March 2016, the ECB announced its intention to start buying corporate bonds directly through the implementation of the corporate sector purchase program (CSPP) as an additional component of the APP (ECB 2016).

Figure 1 shows the ECB net APP purchases, by program.[1] In August 2022, the ECB corporate bond holdings from the CSPP and other collateral monetary policy operations were 344,558 mil. EUR, while the overall APP holdings were 3,262,730 mil. EUR.[2] Thus, around 10.5% of ECB balance sheet is private corporate bonds and, as long as reinvestments in these assets will continue, this amount is expected to remain stable in the next few years (ECB 2022a).

Analogously, the Bank of England (BoE) decided on a number of non-standard monetary policy measures, including the Corporate Bond Purchase Scheme (CBPS or the Scheme), which was launched in August 2016 and further expanded in 2020 (BoE 2021a).

The Federal Reserve (FED), as well, established the Secondary Market Corporate Credit Facility (SMCCF) on March 23, 2020, to support credit to employers by providing liquidity to the market for outstanding corporate bonds (FED 2021).

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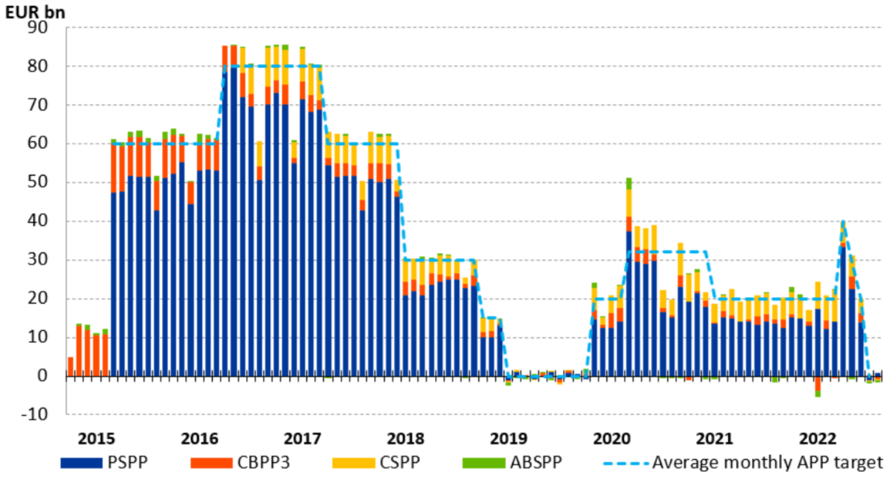
[1] On 9 June 2022 the ECB Governing Council decided to discontinue net asset purchases under the APP as of 1 July 2022. Reinvestments of the principal payments from maturing securities purchased under the programs will continue, in full, for an extended period of time and as long as necessary to maintain ample liquidity conditions and an appropriate monetary policy stance (ECB 2022a).

[2] At amortised cost, in EUR millions, at month-end.

Figure 1: ECB APP net purchases, by program



Note: The average monthly APP targets were first set by the ECB Governing Council at the beginning of the PSPP in March 2015. The additional envelope of €120 billion decided by the Governing Council on 12 March 2020 has been linearised for illustration in this chart, while it will be implemented in full according to the established principles

Source: ECB 2022a

Following these measures, a consistent part of the securities held in the CB portfolios has become bonds of private companies.

The aim of this paper is to shed light on the mechanisms through which a CB can implement a green monetary policy to steer or tilt the allocation of assets and collateral towards low-carbon industries, and reduce the cost of capital for these sectors in comparison to high-carbon ones.

Starting from a corporate bonds purchase program that follows a carbon-neutral monetary policy, we analyze how a shift in the CB portfolio allocation towards bonds issued by low-carbon companies can favor green firms in the market. By means of a *'green monetary policy'* the CB internalizes externalities and public failures deriving from climate change through the inclusion of climate-related risks in the portfolio assessment. The CB operates according to a market efficiency principle, so that the optimal portfolio choice encompasses three objectives: obtaining high returns, containing risks, and reducing firms' environmental footprint.

Finally, we analyze the interactions between the neutral or green CB rebalancing policy and the evolutionary choice (i.e. by means of exponential replicator dynamics) of a population of firms that can decide to be green or not according to bonds borrowing cost.

We obtain some main findings. First, some scenarios are characterized by a strong path dependency in which if a large share of firms employed non-green technology, no investment in green technology occurs in the long run, even if the non-green investment equilibrium is inefficient. We define this equilibrium *technology trap* and show that CSPP monetary policy helps the industry leave the technology trap. Second, green and non-green bond riskiness is a key factor that impacts borrowing costs. The larger the average financial risk of bonds, the lower the share of bonds in the CB portfolio, and the larger the cost. Third, the degree of market competition and of market (im)perfections contribute to amplifying the effects of the green monetary policy by affecting the transmission channel. In the presence of imperfect competition and (or) a high degree of market imperfections the *technology trap* is more likely to happen, and the green monetary policy seems to foster the adoption of green technologies.

The paper is organized as follows. Section 2 provides the institutional background and a short literature review on the issue of greening the monetary policy of central banks. Section 3 first analyses a *'neutral monetary policy'* based on modern portfolio theory (3.1), and then a *'green monetary policy'* by introducing a further CB objective based on the carbon intensity of firms (3.2). The section concludes with a numerical example of the results (3.3). Section 4 studies the interactions between the monetary policy strategy undertaken by the CB and the investment decision of a population of firms based on bond borrowing costs. Section 5 concludes.

2 Literature review and institutional background

Market neutrality has generally been the CB guiding principle of asset purchase programs:[3] the monetary authority buys a proportion of the market portfolio of available corporate and bank bonds (usually investment-grade bonds) to reduce price distortions from their eligible asset purchases[4]. However, this strategy might imply a carbon bias because capital-intensive companies and sectors tend to be more carbon-intensive (Papoutsi, Piazzesi, and Schneider 2021).

The existence of climate externalities requires a reconsideration of market neutrality. In the presence of market failures, adhering to the market neutrality principle may reinforce pre-existing inefficiencies that give rise to a suboptimal allocation of resources. If the market misprices the risks associated with climate change underestimating the social costs of investment, adhering to the market neutrality principle may instead support a market structure that hampers an efficient allocation of resources. In view of such market failures, a market efficiency principle would explicitly recognize that a supposedly 'neutral' market allocation may be suboptimal in the presence of externalities. Indeed, market failures may drive a wedge between market prices on the one hand and efficient asset values that internalize the externalities on the other (Schnabel 2021).

Corporate bond holdings expose CBs to different types of financial risk that might be related to climate change: extreme weather events such as wildfires

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[3] In the ECB case, the operationalisation of this principle entails the monetary authority purchases securities in proportion to their relative market capitalisation (Coeure' 2015).

[4] For example, the Bank of England's Corporate Bond Purchase Scheme (CBPS) follows a principle similar to market neutrality. The CBPS is conducted with the objective of minimizing the impact of asset purchases on the relative borrowing costs across sectors. The principle is implemented via sector key targets, with the potential for deviations (BoE 2021b).

or floods can hit companies' or their customers' premises and destroy their warehouses, manufacturing plants, data centres and supply chains implying additional "physical risk" (Alogoskoufis et al. 2021). In addition, so-called transaction risks result from societal and economic shifts toward a low-carbon and more climate-friendly production model. Such shifts could mean that some sectors of the economy face significant transformations in asset values or higher costs of doing business that alter the value of investments held by banks and insurance companies (Gourdel et al. 2022). For these reasons, some CBs have started to *greener* monetary policy operations to reduce the financial risk related to climate change and to promote a green transition of industries and firms.

On November 5 2021, the Bank of England considered the climate impact of the issuers of bonds within the framework of the CBPS: "with this approach we will incentivize firms to take decisive actions that support an orderly transition to net zero. Purchases will then be tilted or skewed within sectors towards the debt of eligible firms that are performing relatively strongly in support of net zero, and responding most to the incentives we are setting, and away from those who are not" (BoE 2021a, BoE 2021b).

As announced in July 2022, also the Eurosystem aims to gradually decarbonize its corporate bond holdings on a path aligned with the goals of the Paris Agreement. To that end, the ECB will tilt its purchases towards issuers with a better climate performance by reinvesting the sizeable redemptions expected over the coming years. The overall volume of corporate bond purchases will, however, continue to be determined solely by monetary policy considerations and the role played by such purchases in achieving the ECB's inflation target (ECB 2022b). The ECB has also announced that when government and corporate bonds come to maturity in the context of its QE program, new bonds will be bought in the market to keep the money stock (money base) unchanged. This creates a 'window of opportunities' for the ECB. It could replace the old bonds with new 'environmental bonds' over time to establish a well-diversified portfolio that also includes the value and the risk profile of climate change and carbon transition effects (Grauwe 2019).

Therefore, the objective of a green monetary policy is to steer or tilt the allocation of assets and collateral toward low-carbon sectors and firms. This could reduce the cost of capital for those companies and sectors in comparison to high-emission industries. The allocation policy must be designed and executed so that it does not interfere with the effective implementation of monetary policy and the transmission mechanism. Price stability is and should remain the top priority for central banks.

In this paper, we fix the dimension of the corporate bonds purchase program (i.e. the overall CB demand of private bonds), and focus on the composition of the CB balance sheet between two typologies of corporate bonds: green and non-green bonds. We study how steering the CB eligibility criteria towards low-carbon bonds issued by environmentally friendly companies, following the market efficiency principle, can help the financing condition, favoring green companies in the market.

3 The Model

Equation (1) shows the total amount of corporate bonds in an economy, eligible[5] for a CB purchase program (B_T) given by green corporate bonds B_G issued by companies to finance environmentally sustainable projects, and non-green/conventional corporate bonds B_N issued by firms for investment that are not related to emission or pollution abatement technologies:

$$B_T = B_G + B_N \tag{1}$$

We define the share of green bonds $x = \frac{B_G}{B_T}$, and the complementary share of non-green bonds $1 - x = \frac{B_N}{B_T}$ in the economy.

For simplicity, we assume that the CB can identify the type of bond without ambiguity. While the assumption does not alter the conclusions of the paper, it avoids dealing with various criteria that are often different for each type of institution and/or asset purchase program under consideration, since no international standard has been established yet (OECD 2017 and see for a taxonomy, Commission 2020)[6].

If green and conventional bonds were perfect substitutes for banks, production and investment in both sectors would not be affected (Ferrari and Landi 2021) after the CB tilts the portfolio composition towards green bonds and keeps the total assets constant. However, green and non-green bonds signal two different types of use of the financial resources and hence, are imperfect substitutes both for the issuing firms and for investors (Flammer 2021, Zerbib 2019, Gianfrate and Peri 2019). We, therefore, model both types using two distinct supply functions. The aggregate supply of corporate green bonds in the market negatively depends on green bond yield: $B_G = f(\mu_G)$. Indeed, when the interest rate on this specific category of bonds (μ_G) increases, the firms' relative supply of bonds decreases because it becomes more costly for companies to finance sustainable-friendly projects through the issuance of green bonds. The aggregate supply function is modeled by means of the unitary isoelastic function given by eq. (2a). Similarly, the green bond supply in terms of share $x(\mu_G)$ is

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[5] The bond and issuer eligibility conditions set forth by the European Central Bank can be found in ECB 2016, Zaghini 2019.

[6] The Eurosystem has developed a climate scoring methodology to assess the climate performance of eligible issuers that is based on three sub-scores: (i) backward-looking climate metrics, in the form of (disclosed) past GHG emissions and emission intensities (normalised by revenue); (ii) forward-looking climate metrics, such as whether the issuer has credible and ambitious decarbonization targets in place; and (iii) the quality of climate disclosures, such as their completeness and their verification by third parties. These metrics are based on publicly available data as well as other relevant information and methodologies, such as science-based targets, etc. (ECB 2022c).

given by eq. (2b), the inverse supply function $\mu_G(x)$ is (2c):

$$B_G(\mu_G) = \frac{\alpha}{\mu_G} \Leftrightarrow \tag{2a}$$

$$x(\mu_G) = \frac{\alpha}{\mu_G\, B_T} \Leftrightarrow \tag{2b}$$

$$\mu_G(x) = \frac{\alpha}{x\, B_T} \tag{2c}$$

Analogously, the aggregate supply of corporate non-green bonds in the market negatively depends on non-green bonds yield: $B_N = f(\mu_N)$. This aggregate supply function is unitary isoelastic, and given by eq. (3a). The equivalent non-green bonds supply in terms of share $1 - x(\mu_N)$ is eq. (3b), as well as the inverse supply function $\mu_N(1 - x)$ is (3c):

$$B_N(\mu_N) = \frac{\beta}{\mu_N} \Leftrightarrow \tag{3a}$$

$$1 - x(\mu_N) = \frac{\beta}{\mu_N\, B_T} \Leftrightarrow \tag{3b}$$

$$\mu_N(1 - x) = \frac{\beta}{(1 - x)\, B_T} \tag{3c}$$

By definition, the total amount of corporate bonds in the economy as well as the yield on bonds must be positive ($B_T, \mu_G, \mu_N > 0$), it follows from eqs. (2c) and (3c) that also $\alpha, \beta > 0$. The parameters α and β are scaling factors of the aggregate supplies of green and non-green bonds respectively, proxy of the relative market size of the two types of bonds considered.

3.1 Neutral monetary policy

The total volume of corporate bonds purchased by the CB through a large-scale purchase program is only determined by monetary policy considerations, i.e. inflation targeting (Bacchiocchi and Giombini 2021), thus, we assume that the representative CB is the only corporate bonds investor in the economy and acquires the total amount of eligible bonds in the economy[7]. Therefore, we focus only on the relative composition (i.e green or non-green) of purchase program B_T and study the impact of a CB strategy that includes environmental considerations (i.e. *green monetary policy*), to study the occurrence of portfolio re-balance and its effect on the cost of bonds for firms.

Based on modern portfolio theory (Bodie, Kane, and Marcus 2021), the CB considers the average expected yields of green μ_G and non-green bonds μ_N, their average volatility (i.e., the standard deviation of their returns), given respectively by $\sigma_G, \sigma_N > 0$, and the covariance between the two types of corporate bonds $\sigma_{G,N}$[8]. The covariance $\sigma_{G,N}$ is related to the correlation coefficient

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[7] This holds without loss of generality when there are no spillovers between the CB and other corporate bonds investors.

[8] To use standard deviations we assume that returns are normally distributed and that the CB, as an investor, has access to sufficient information to evaluate these variables.

$r_{G,N} = \frac{\sigma_{G,N}}{\sigma_G\,\sigma_N}$, which, to be economically meaningful, must range between -1 (i.e. perfect negative correlation) and $+1$ (i.e. perfect positive correlation). Thus, it holds that:

$$-1 \leq \frac{\sigma_{G,N}}{\sigma_G\,\sigma_N} \leq 1 \tag{4}$$

According to the capital asset pricing model (CAPM), the CB portfolio expected yield $\mu_P(x)$ is a convex combination of the individual yields, where the weights are the share of green bonds $x \in (0,1)$ and non-green bonds $1-x$ (i.e. the complementary part) in the CB portfolio and in the market:

$$\mu_P(x) = x\,\mu_G + (1-x)\,\mu_N \tag{5}$$

Substituting the inverse supply functions of green (2c) and non-green bonds (3c) into eq. (5), and defining the CB portfolio's expected variance $\sigma_P^2(x)$, based on the volatility (i.e. standard deviation) $\sigma_i > 0, i = G, N$, and the covariance $\sigma_{G,N}$ of the individual type of bonds, we obtain:

$$\begin{cases} \mu_P(x) = \frac{\alpha}{B_T} + \frac{\beta}{B_T} \\ \sigma_P^2(x) = x^2\,\sigma_G^2 + (1-x)^2\,\sigma_N^2 + 2\,x\,(1-x)\,\sigma_{G,N} \end{cases} \tag{6}$$

The system of equations in (6) determines a tuple of points, i.e. the expected yield and expected variance of the portfolio, in relation to share x. It describes the mean-variance trade-off that the CB faces for all the possible combinations/allocations of green (x) and non-green $(1-x)$ bonds[9]. Consequently, corporate bonds come in a variety of risk-reward levels depending on the issuing company's creditworthiness. While the CB prefers assets that have the highest expected return, it also seeks to minimize uncertainty about corporate bonds future return. We assume that the CB chooses the combination of green and non-green bonds with the optimal risk-reward level, i.e. the portfolio allocation that offers the maximum return-to-risk ratio, i.e. the optimal portfolio x^* in the CAPM. The CB risk-adverse preference function in a *neutral monetary policy* setup can be formalised as a capital allocation line defined by the following (7):

$$\mu_P(x) = r_F + S_P\,\sigma_P(x) \tag{7}$$

The CB maximizes the portfolio return $\mu_P(x)$ for a given portfolio risk $\sigma_P(x)$, where S_P is the Sharpe ratio or reward-to-risk ratio (Sharpe 1971), and $r_F \geq 0$ is the equivalent risk-free asset (i.e. the yield associated to a risk-free asset, for example a short-term U.S. treasury bond). Equation (7) shows the trade-off between the expected portfolio return $\mu_P(x)$ and its volatility $\sigma_P(x)$ and thus defines the risk-aversion preference of the CB. The CB is willing to hold a riskier portfolio if and only if it guarantees a higher average return reflected in S_P. Therefore, the CB maximizes the reward-to-risk ratio S_P given the constraints

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[9] The efficient frontier is the set of portfolios which satisfy the condition that no other portfolio exists with a higher expected return but with the same standard deviation of return (i.e., the risk).

in (6) by determining the share x that maximizes the Sharpe ratio of a portfolio that is on the envelope of the Markowitz bullet (Markowitz 1952):[10]

$$\begin{aligned} \max_{x} \quad S_P = \frac{\mu_P(x) - r_F}{\sigma_P(x)} \quad \text{s.t.} \\ \text{constraints in (6)} \end{aligned} \tag{8}$$

Note that $\mu_P(x) \geq r_F$ in (8) requires that:

$$\frac{\alpha + \beta}{B_T} \geq r_F \tag{9}$$

From the Sharpe ratio condition (8), it is also required that $\sigma_P^2(x) > 0$ in (6). It must therefore hold that:

$$\sigma_{G,N} > -\frac{x\sigma_G^2}{2(1-x)} - \frac{(1-x)\sigma_N^2}{2x} \tag{10}$$

The problem in (8) can be reduced to solving the unconstrained maximization problem

$$\max_{x} \quad \frac{\frac{\alpha}{B_T} + \frac{\beta}{B_T} - r_F}{\sqrt{x_G^2 \, \sigma_G^2 + (1-x)^2 \; \sigma_N^2 + 2\,x\;(1-x)\;\sigma_{G,N}}} \tag{11}$$

The solutions to problem (11) returns the optimal shares of green and non-green corporate bonds in the CB portfolio and thus in the market, given by:

$$x^* = \frac{\sigma_N^2 - \sigma_{G,N}}{\sigma_G^2 + \sigma_N^2 - 2\,\sigma_{G,N}} \tag{12a}$$

$$1 - x^* = \frac{\sigma_G^2 - \sigma_{G,N}}{\sigma_G^2 + \sigma_N^2 - 2\,\sigma_{G,N}} \tag{12b}$$

From condition (4) and given that (12a), (12b) $\in (0,1)$, it must hold:

$$\sigma_N^2 > \sigma_{G,N} \tag{13a}$$

$$\sigma_G^2 > \sigma_{G,N} \tag{13b}$$

In the following, we define the derivatives of the optimal shares (12a), (12b)

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[10] Graphically, the slope of the optimal set, the maximum Sharpe ratio, is such that it is tangent to the portfolio efficient frontier (Sharpe 1971).

with respect to the model parameters:

$$\frac{\partial x^*}{\partial \sigma_N^2} = \frac{\sigma_G^2 - \sigma_{G,N}}{\left(\sigma_G^2 - 2\sigma_{G,N} + \sigma_N^2\right)^2} > 0 \tag{14a}$$

$$\frac{\partial x^*}{\partial \sigma_G^2} = \frac{\sigma_{G,N} - \sigma_N^2}{\left(\sigma_G^2 - 2\sigma_{G,N} + \sigma_N^2\right)^2} < 0 \tag{14b}$$

$$\frac{\partial x^*}{\partial \sigma_{G,N}} = \frac{\sigma_N^2 - \sigma_G^2}{\left(\sigma_G^2 - 2\sigma_{G,N} + \sigma_N^2\right)^2} \gtreqless 0 \tag{14c}$$

$$\frac{\partial^2 x^*}{\partial \sigma_N^2 \partial \sigma_G^2} = \frac{\sigma_N^2 - \sigma_G^2}{\left(\sigma_G^2 - 2\sigma_{G,N} + \sigma_N^2\right)^3} \gtreqless 0 \tag{14d}$$

$$\frac{\partial^2 x^*}{\partial \sigma_G^2 \partial \sigma_{G,N}} = \frac{2\sigma_{G,N} + \sigma_G^2 - 3\sigma_N^2}{\left(\sigma_G^2 - 2\sigma_{G,N} + \sigma_N^2\right)^3} \gtreqless 0 \tag{14e}$$

$$\frac{\partial^2 x^*}{\partial \sigma_N^2 \partial \sigma_{G,N}} = -\frac{2\sigma_{G,N} - 3\sigma_G^2 + \sigma_N^2}{\left(\sigma_G^2 - 2\sigma_{G,N} + \sigma_N^2\right)^3} \gtreqless 0 \tag{14f}$$

As expected, an increase of the variance (i.e. financial risk) reduces the optimal share of the correspondent corporate bond in CB portfolio, while the effect of the covariance on x^* can be positive, negative or null, depending on the difference of the two variances.

Given the optimal shares, it is possible to retrieve the optimal amount of green B_G^* and non-green bonds B_N^* in the market:

$$B_G^* = x^* B_T \tag{15a}$$

$$B_N^* = (1 - x)^* B_T \tag{15b}$$

Substituting the optimal portfolio amount of green and non-green bonds into the aggregate inverse supply functions (2c) and (3c), provides the equilibrium bonds yields μ_G^* and μ_N^*:

$$\mu_G^* = \frac{\alpha}{B_G^*} = \frac{\alpha\,(\sigma_G^2 + \sigma_N^2 - 2\,\sigma_{G,N})}{B_T\,(\sigma_N^2 - \sigma_{G,N})} \tag{16a}$$

$$\mu_N^* = \frac{\beta}{B_N^*} = \frac{\beta\,(\sigma_G^2 + \sigma_N^2 - 2\,\sigma_{G,N})}{B_T\,(\sigma_G^2 - \sigma_{G,N})} \tag{16b}$$

These bond yields represent the cost of capital for each type of firms issuing the bond. Given eq. (16a), (16b), the monetary authority can reduce the yield/cost of capital for green companies and increase the yield/cost of capital for non-green firms by altering the composition x^* of its balance sheet without modify the latter's total dimension (B_T).

3.2 Green monetary policy

The existence of climate externalities, and physical and transitional risks related to climate change question market neutrality, as it could reinforce pre-existing inefficiencies that give rise to erroneous prices and suboptimal resources allocation. The objective of the *green monetary policy* is to internalize such externalities and risks to obtain an efficient allocation of financial resources that take into consideration climate related issues.

In other words, the CB desires to re-balance its portfolio to reduce the cost of capital for firms that invest in sustainable/green projects, while fixing, at the same time, the overall dimension of the balance sheet B_T.

By increasing the relative share x^* of green bonds, the CB reduces the borrowing cost for environmental sustainable firms and it renders more costly for companies to finance non-green investment projects. This *green monetary policy* should encourage firms to invest and shift to an environmental sustainable production. We model the green monetary policy by introducing a steering/tilting factor (Schoenmaker 2021) that governs the CB's portfolio:

$$p = \frac{C_N}{C_G} \tag{17}$$

where $C_i, i = G, N$ is a synthetic indicator of the environmental footprint of the i-type issuer, e.g. the average carbon emissions and/or other environmental measures. Note that the average environmental footprint indicator of non-green issuers C_N is greater than the same indicator for green issuers C_G. This is consistent with studies such as Fatica, Panzica, and Rancan 2021, where green bond issued by non-financial corporations are associated with a reduction in firm-level carbon emissions induced by climate friendly investment projects.

Since the tilting factor p in eq. (17) is the ratio between the two footprint indicators, it always exceeds 1. Moreover, this ratio defines the extent of the greening monetary policy and accounts for the additional risks (physical, transitional) related to the carbon footprint of firms that issue corporate bonds to finance non-sustainable investment. Since these projects (linked to conventional bonds) are not green, they: (1) are more exposed to adverse climatic events and natural disasters that bring direct and indirect physical assets damages (e.g. business disruption, system failures, disruption of transportation facilities and telecommunications infrastructure, etc.), (2) are more vulnerable to an increasing legal and regulatory environmental-friendly framework where compliance risk as well as litigation and liability costs associated with climate-sensitive investments undermine business profitability, (3) become target of economic policy that demand a reduction in the use of fossil fuels and carbon emission (e.g. carbon tax) (Alogoskoufis et al. 2021, ECB/ESRB 2021).

The climate-related risks become relevant and are internalized via the CB corporate bond purchase program. As they affect the variance of the corresponding bonds (σ_N^2), we define a modified variance $\hat{\sigma_N}^2$ that considers beside the financial risk, these climate-related risks:

$$\hat{\sigma_N}^2 = p\ \sigma_N^2 \tag{18}$$

given that the tilting/steering factor $p > 1$, the overall risk of non-green corporate bonds increases[11]. In this way, the CB internalizes the externalities

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[11] Note that the case of *neutral monetary policy*, is obviously the special case in which $p = 1$.

and public failures through the inclusion of climate-related risks in the portfolio assessment. Therefore, following the market efficiency principle, the optimal portfolio choice in a *green monetary policy* setting encompasses three objectives: obtaining high returns, containing risk/volatility, and reducing firms' environmental footprint, defined by:

$$
\begin{aligned}
&\max_x s_P = \frac{\mu_P(x) - r_F}{\sigma_P(x)} \quad \text{s.t.} \\
&\begin{cases} \mu_P(x) &= \frac{\alpha}{B_T} + \frac{\beta}{B_T} \\ \sigma_P^2(x) &= x^2\,\sigma_G^2 + (1-x)^2\ \hat{\sigma_N}^2 + 2\,x\ (1-x)\ \sigma_{G,N} \end{cases}
\end{aligned}
\tag{19}
$$

and the corresponding solutions in (12a) and (12b) with the substitution of $\hat{\sigma_N}^2$ in eq. (18).
Since

$$
\frac{\partial x^*}{\partial p} = \frac{\sigma_N^2\left(\sigma_G^2 - \sigma_{G,N}\right)}{\left(\sigma_G^2 + p\,\sigma_N^2 - 2\,\sigma_{G,N}\right)^2} > 0 \tag{20}
$$

from condition (13b), the CB optimal portfolio contains a higher share of green bonds x^* and a lower share of non-green bonds $1 - x^*$. The optimal amount of the two types of bonds B_G^* and B_N^* is given by eqs. (15a) and (15b), the bonds yields μ_G^* and μ_N^* are given by (16a) and (16b) after substituting $\hat{\sigma_N}^2$ in (18):

$$
\mu_G^* = \frac{\alpha}{B_G^*} = \frac{\alpha\,(\sigma_G^2 + \hat{\sigma_N}^2 - 2\,\sigma_{G,N})}{B_T\,(\hat{\sigma_N}^2 - \sigma_{G,N})} \tag{21a}
$$

$$
\mu_N^* = \frac{\beta}{B_N^*} = \frac{\beta\,(\sigma_G^2 + \hat{\sigma_N}^2 - 2\,\sigma_{G,N})}{B_T\,(\sigma_G^2 - \sigma_{G,N})} \tag{21b}
$$

The CB lowers the financing costs for environmentally sustainable firms and tightens the financing conditions of non-green companies, i.e. increasing the so-called green premium or *greenium* (E. Agliardi and R. Agliardi 2021, Caramichael and Rapp 2022), as

$$
\begin{aligned}
\frac{\partial \mu_G^*}{\partial p} &= \frac{\alpha\,\sigma_N^2\left(\sigma_{G,N} - \sigma_G^2\right)}{B_T\left(\sigma_{G,N} - p\,\sigma_N^2\right)^2} < 0 \\
\frac{\partial \mu_N^*}{\partial p} &= -\frac{\beta\,\sigma_N^2}{B_T\left(\sigma_{G,N} - \sigma_G^2\right)} > 0
\end{aligned}
\tag{22}
$$

A short numerical example shows the impact of a *green monetary policy* CSPP undertaken by a representative CB. In the economy, a volume of eligible corporate bonds equal to $B_T = 140,000$ millions EUR or USD is acquired by the Central Bank through the CSPP. The scaling factors of the aggregate bonds supply are $\alpha = 2300$ for green bonds, and $\beta = 4000$ for non-green bonds. Furthermore, the CB can observe the yields trend to assess the financial risk related to these assets. The volatility, given by the standard deviation, of green bonds

$\sigma_G = 0.20$ is higher than that of non-green bonds $\sigma_N = 0.15$, and covariance between the two types of bonds is $\sigma_{G,N} = -0.002$, corresponding to a moderate negative correlation coefficient $r_{G,N} = -0.067$. The risk-free asset has a yield of $r_F = 0.02$. The assumptions satisfy conditions (4), (9), (10), (13), and Table 1 compares the optimal shares, amounts and yields of green and non-green bonds for a *neutral monetary policy* ($p = 1$) and for a *green monetary policy* ($p = 1.1$).

Table 1: Comparison between neutral and green monetary policy

Type of mon. pol. (p)	x^*	$1 - x^*$	B_G^*	B_N^*	μ_G^*	μ_N^*
Neutral ($p = 1$)	36.8%	63.2%	$51,579$	$88,421$	4.46%	4.52%
Green ($p = 1.1$)	40.9%	59.1%	$54,473$	$85,527$	4.22%	4.68%

Table 1 shows that if the tilting factor $p > 1$, that is, as long as the CB accounts for the additional risks related to the carbon footprint of firms that issue corporate bonds to finance non-sustainable investment, the financing conditions of green firms improve, *ceteris paribus*.

4 Green monetary policy and firm investment choice

In this section, we consider the interaction between monetary policies, i.e. neutral or green, and the investment choice of firms in a given sector.

The investment survey of the European Investment Bank (EIB) in Figure 2 shows that an increasing number of firms is investing in green/climate-related measures (EIB 2022)[12].

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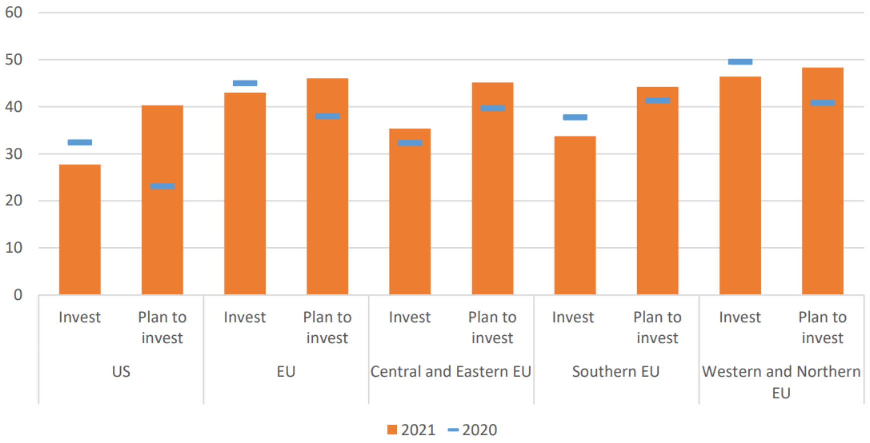
[12] The share of firms investing in climate measures in 2021 is marginally below the share in 2020, which is likely the result of the repercussions the COVID-19 pandemic had on firms' investment plans. Overall, the share of EU firms investing in climate-related measures is significantly higher than in the United States, with companies in Western and Northern Europe leading the trend (EIB 2022).

Figure 2: Firms (in %) investing or planning to invest in climate-related measures



Source: EIBIS 2021, EIBIS 2020

Note: The base is all firms (data not shown for those who said do not know/refused to answer)

Question: Has your company already invested to tackle the impacts of weather events and reduce carbon emissions?

Furthermore, Europe has also become a world leader in the issuance of green bonds. In late 2021, the volumes issued by companies and national and sub-national governments in the EU-27 reached €497 bn compared to a bond volume of non-European issuers at around €558 bn (Fatica and Panzica 2021).

Building on this evidence and similar to Pindyck 1988, 1991, we model the potential impact of a CSPP program on a population of firms which invests capital $C(t)$ in each period t. The population of firms belongs to an industry with two technologies of production: a green technology G and a non-green technology N. Consequently, the firms in the sector can invest capital $C(t)$ at every period t (e.g. every year) in either green/climate-related technology $G(t)$ (i.e. ’green investment’) or in non-green technology $N(t)$ (i.e. ’non-green investment’). The share of green investment in the industry is $0 \leq y(t) = \frac{G(t)}{C(t)} \leq 1$ and the complementary share of non-green investment is $1 - y(t) = \frac{N(t)}{C(t)}$, assuming that the background growth rate of bond capital $r(t)$ is independent of the technology investment choice $i = G(t), N(t)$ at each time t.

We assume that firms make investment choices under limited information: firms do not know exactly what the return on investment of each technology will be and/or are not able to compute the optimal alternative following traditional profit maximization rules. In this case, the decision cannot be based on expected return on investment as in a perfect information setting. Instead, firms imitate the investment behavior of other firms. More specifically, each company in the industry simply observes a small subset of other firms and replicates the investment strategy of the most successful ones.

Similar to Shaffer 1991, and Calcagnini, Gardini, et al. 2022, we assume that the firm investment on technology $i(t)$ earns a marginal return $MR_i(t)$:

$$MR_G(t) = a_G - b_G\, y(t) \tag{23a}$$

$$MR_N(t) = a_N - b_N\,[1 - y(t)] \tag{23b}$$

where the parameters $a_G, a_N, b_G, b_N > 0$ depend on the characteristics of the manufacturing technology i of the sector and are assumed to be constant in time[13]. The total earnings $E_i(t)$ from a given technology investment/adoption $i(t)$ are the integral of (23a),(23b) with respect to the correspondent investment, i.e.:

$$E_G(t) = a_G\, y(t) - \frac{b_G}{2}\, y(t)^2 \tag{24a}$$

$$E_N(t) = a_N\,[1 - y(t)] - \frac{b_N}{2}\,[1 - y(t)]^2 \tag{24b}$$

Given a relatively small firm size, firms are price-taker in the bonds market. At each time t, firms can issue either a green bond at a constant interest rate μ_G^* to finance the investment in the green technology G, or they can issue non-green bonds at a constant interest rate μ_N^* to finance the investment in non-green technology N[14]. The cost of the two alternative types of bonds is determined by the portfolio optimization problem of the monetary authority in relation to its policy and defined by (21). For the sake of simplicity, both types of bonds have the same maturity. As a result, the borrowing cost of a firm is given by the principal amount to be reimbursed at maturity, which coincides with the value of the investment, and the (fixed) interest rate μ_G^* or μ_N^* on this debt,[15]

$$C_G(t) = y(t)\,(\mu_G^* + 1) \tag{25a}$$

$$C_N(t) = [1 - y(t)]\,(\mu_N^* + 1) \tag{25b}$$

Considering both the total earnings from the investment (24a), (24b) and the corporate bond cost (25a), (25b), we define the firms return on green investment $\pi_G(y)$ as a function of green investment in the industry at time t, and the firms return on non-green investment $\pi_N(1-y)$ as a function of non-green investment at time t[16],

$$\pi_G\,(y) = a_G\, y - \frac{b_G}{2}\, y^2 - (\mu_G^* + 1)\, y \tag{26a}$$

$$\pi_N\,(1-y) = a_N\;(1-y) - \frac{b_N}{2}\;(1-y)^2 - (\mu_N^* + 1)\;(1-y) \tag{26b}$$

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[13] For this reason we can refer to them as *structural parameters*.

[14] Here we do not consider the phenomenon of green-washing, in which some firms issue green bonds to bear a lower financing cost employing the proceeds in non-green investment.

[15] Since the maturity of green and non-green corporate bonds is the same, it is sufficient to compare firm' borrowing cost in only one period of time.

[16] For sake of brevity we omit t in eqs. (26a), (26b).

The CB corporate bonds purchase program can follow the *neutral monetary policy* or the *green monetary policy* framework. The type of program affects the relative bonds' cost μ_G^* and μ_N^* (in eqs. (21a),(21b)), and therefore, the firms' decision to invest in environmental-friendly technology.
The decision of the firms to invest in the green technology $y \in [0, 1]$ is assumed to evolve in discrete time, according to an exponential replicator dynamics R, as in Cabrales and Sobel 1992, Bacchiocchi and Bischi 2022:

$$y\,(t+1) = f\,(y\,(t)) = (1-\eta)\; y\,(t) + \eta\, \frac{y\,(t)}{y\,(t) + (1 - y\,(t))\; e^{\,-\gamma\; g(y(t))}} \tag{27}$$

The dynamic model (27) describes the time evolution of the green investment by introducing adaptive adjustments based on a direct comparison of the expected firm's return on investment:

$$g\,(y\,(t)) = \pi_G\,(y\,(t)) - \pi_N\,(1 - y\,(t)) \tag{28}$$

According to (27) and (28), at each discrete time t, the share of green investment y increases (decreases) in $t+1$ when a firm's return in green investments is expected to be higher (lower) than the return on non-green investments. The parameter $\gamma > 0$ represents the speed of technology adoption and expresses the firms' ability and propensity to switch to the alternative manufacturing technological as a profit gain is observed in the current time period. The velocity of technology adoption is strictly related to adjustment costs and the irreversibility of investment, and a lower value of γ indicates a slower speed of adoption[17]. Equation (27) also captures the level of inertia as a consequence of the degree of competitiveness between firms, measured by the parameter $0 \leq \eta \leq 1$. For $\eta \to 0$ the firms of the industry have the highest degree of inertia. In this case, investment choices do not change over time, since $y(t+1) = y(t) = y(0)$; while for $\eta \to 1$, no anchoring exits since a firm's survival critically depends on quickly adopting the most profitable technology of production, i.e. $y(t) \to 1$ if $g(y) > 0$ and $y(t) \to 0$ if $g(y) < 0$.

4.1 Analysis

Since $y(0) \in [0; 1]$ then $y(t) \in [0; 1]$ for each $t \geq 0$, as it follows from the inequality $0 \leq \frac{y}{y+(1-y)\,e^{-\gamma\, g(y)}} \leq 1$. Additionally, it is straightforward to see that two pure fixed points exist at $y^* = 0$ and $y^* = 1$ (i.e. *pure equilibria*), where "all firms invest in non-green technology N" and "all firms invest in green technology G", respectively. The interior fixed points (i.e. *mixed equilibria*) are then given by the solution to $g(y^*) = 0$ in (28). Solving for $\pi_G = \pi_N$ with

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[17] It is determined by whether once installed capital has little or no value unless used in production (Bertola 1998), its industry or firm-specificity (Pindyck 1991), and as a consequence its intangibility, the difficulty of re-employment, market imperfections (Calcagnini, Giombini, and Travaglini 2019).

respect to y, we obtain the position of the interior fixed points:[18]

$$y^*_{1,2} = \frac{c \pm \sqrt{c^2 - 4d\left(1 - a_N + \frac{b_N}{2} + \mu^*_N\right)}}{2d} \tag{29a}$$

$$\text{where } c = 2 - a_G - a_N + b_N + \mu^*_G + \mu^*_N \tag{29b}$$

$$d = \frac{1}{2}\left(b_N - b_G\right) \tag{29c}$$

where μ^*_G and μ^*_N are given by (21a) and (21b) respectively.

Two interior fixed points exist if and only if $0 < y^*_{1,2} < 1$ and the discriminant $\Delta = c^2 - 4d\left(1 - a_N + \frac{b_N}{2} + \mu^*_N\right) > 0$.

The asymptotic stability of the fixed points in discrete time is given by the following condition: $-1 < R'(y^*) < 1$, where $R'(y^*)$ is the derivative of (27) at fixed point y^* [19]. The derivatives $R'(y^*)$ at each of the four fixed points are:

$$R'(0) = 1 - \eta\left(1 - e^{\gamma\left(1 - a_N + \frac{b_N}{2} + \mu^*_N\right)}\right) \tag{30}$$

$$R'(1) = 1 - \eta\left(1 - e^{\gamma\left(1 - a_G + \frac{b_G}{2} + \mu^*_G\right)}\right) \tag{31}$$

$$R'(y^*_1) = 1 - \frac{\gamma\eta r(r - c)(c - 2d - r)}{4d^2} \tag{32a}$$

$$\text{with } r = \sqrt{\left(b_G - b_N\right)\left(2 - 2a_N + b_N + \mu_N\right) + c^2} \tag{32b}$$

$$R'(y^*_2) = 1 - \frac{\gamma\eta r(r + c)(c - 2d + r)}{4d^2} \tag{33a}$$

where μ^*_G and μ^*_N are given by (21a) and (21b) respectively.

Given the complexity of the derivatives, we cannot derive analytical conditions in terms of the model parameters. We therefore explore numerically the dynamical proprieties of the system (27) when parameters change to infer relevant economic implications.

In particular, we will define four scenarios with at least one internal fixed point [20] for different values of the *structural parameters* that define the characteristic of the manufacturing technology $i = G, N$ of the industry: a_G, a_N, b_G, b_N.

---

[18] Since (26a) and (26b) are second degree polynomials, only none, one or two interior fixed points exist.

[19] The stability condition includes both an upper and a lower threshold for the slope of the non-linear function R at the equilibrium point, and the two limiting values -1 and $+1$ constitute two different conditions of non-hyperbolicity of the fixed point. When the condition of non-hyperbolicity $R'(y^*) = 1$ is crossed, as parameters vary, potentially three bifurcations can occur: fold, transcritical (or stability exchange) and pitchfork bifurcation. The bifurcation occurring at $R'(y^*) = -1$ is denoted as flip, at which the fixed point changes its oscillatory stability (i.e. convergence through damped oscillations) into oscillatory instability (i.e. trajectories starting close to y^* exhibit oscillatory expansion).

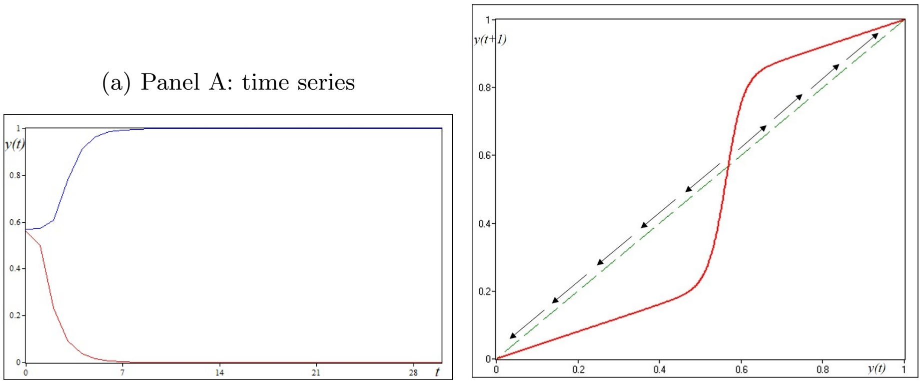
[20] We will ignore those scenarios in which only *pure equilibria* exist, i.e. firms invest fully in green or non-green technology independently from the starting conditions.

We take the parameter values in Table 1 as a benchmark case for a *neutral monetary policy* setting and investigate how a change in p influences the share of green and non-green investment in the industry.

4.2 Unstable internal equilibrium and path dependency

We start with the easiest scenario in which one internal unstable fixed point exists at $y_1^* = 0.569$ ($R'(y_1^*) = 7.30$). The pure equilibria at $y^* = 0$, ($R'(0) = 0.40$) and $y^* = 1$ ($R'(1) = 0.40$) are stable. The time series plot in Figure 3a shows that the interior equilibrium is a separatrix and defines the basins of attraction of the two attracting pure equilibria. Starting from the initial condition (*i.c.*) 0.56 at which 56% of the investment in the industry are in green technology and the remainder of 44% are in the conventional non-green technology, the time series in red, given by (27), converges to $y^* = 0$, i.e. all the firms of the sector eventually invest in non-green technology in the long-run. This holds for all $i.c. < y_1^*$ as highlighted by the arrows in the phase plot of Figure 3b. For all $i.c. > y_1^*$ (such as $i.c. = 0.57$ of the blue time series in Fig. 3a), R converges to $y^* = 1$, i.e. all the companies invest in green technology after a certain period of time t.

Figure 3: Scenario of unstable internal equilibrium $y_1^* = 0.569$

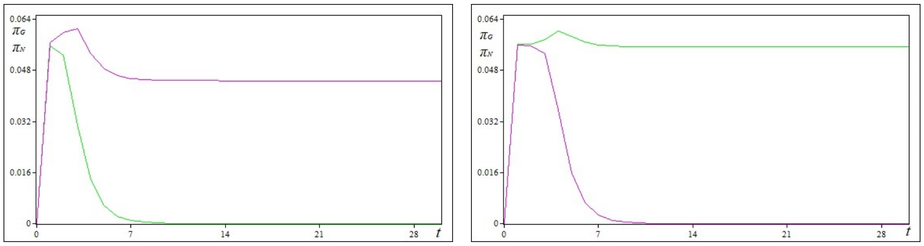

(b) Panel B: phase plot

(a) Panel A: time series

Parameters: $a_G = 1.2, a_N = 1.24, b_G = 0.2, b_N = 0.3, \eta = 0.6, \gamma = 400, \alpha = 2300, \beta = 4000, BT = 140000, \sigma_G = 0.2, \sigma_N = 0.15, \sigma_{G,N} = -0.002, p = 1, r_F = 0.02$. In panel A, for the red time series the initial condition (*i.c.*) is 0.56, for the blue time series $i.c. = 0.57$.

In the former case, in Fig. 4a, profits from green investment $\pi_G = 0$, while the non-green investment generates an equilibrium profit $\pi_N = 0.046$. Fig. 4b shows the latter case in which green investment leads to a profit $\pi_G = 0.055$, and non-green profits are $\pi_N = 0$ in the long-run.

Figure 4: Profits' evolution



(a) Panel A: Convergence to $y^* = 0$

(b) Panel B: Convergence to $y^* = 1$

Parameters: same parameters of Figure 3. In panel A *i.c.* $= 0.56$, in panel B *i.c.* $= 0.57$. The green curve represents green profit π_G, the pink curve non-green profit π_N.

This scenario is characterized by a strong path dependency: if a large share of firms employed non green technology, no investment in green technology occurs in the long-run, while if a critical share of the firms invests in green technology, eventually the entire firm population will adopt the latter technology. Furthermore, note that the all non-green investment equilibrium is Pareto sub-optimal in terms of profits compared to the all green investment equilibrium (i.e. $0.046 < 0.055$). This constitutes a *technology trap*, where all the firms in the sector are stuck with a sub-optimal choice.

CSPP monetary policy can be used to help the industry leave *technology trap*. This is demonstrated by the bifurcation diagram[21] for parameter p of Figure 5. In the previous scenario, the CB ran a neutral monetary policy (i.e. $p = 1$). By increasing p, the monetary authority moves towards a green monetary policy reducing the cost of corporate green bonds. Consequently, increasing p shifts the internal equilibrium and increases the basin of attraction of the full green investment. At *i.c.* $= 0.56$, a value of $p = 1.04$ leads to a convergence towards the all green investment equilibrium. For higher p values, lower initial conditions converge to the same equilibrium.

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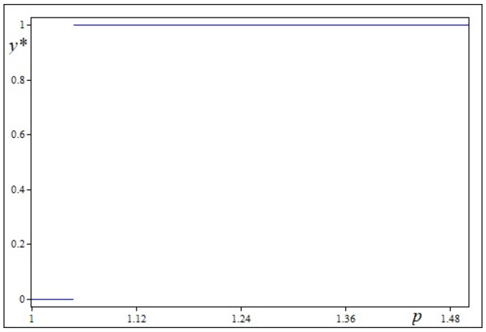
[21] In dynamical systems, a bifurcation diagram shows the values visited or approached asymptotically (fixed points, periodic orbits, or chaotic attractors) of a system as a function of a bifurcation parameter in the system.

Figure 5: Bifurcation diagram for p

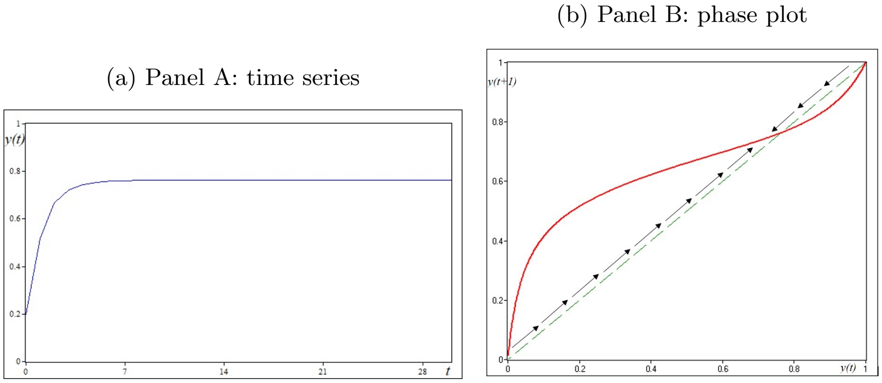


Parameters: same parameters of Figure 3. The *i.c.* $= 0.56$.

4.3 Stable internal equilibrium and transition to deterministic chaos

We consider the case with only one internal equilibrium $y_1^* = 0.763$, which is stable ($R'(y_1^*) = 0.47$). The two pure equilibria are unstable ($R'(0) = 12.60, R'(1) = 2.45$). Figure 6 highlights the evolution in time (6a) of the green investment share starting from *i.c.* $= 0.2$. The firm population converges to $y_1^* = 0.76$ (i.e. 76% green technology, 24% non-green technology adoption in the sector). In this case y_1^* is the unique global attractor of the system and is reached for every $0 < i.c. < 1$ (Fig. 6b).

Figure 6: Scenario of unique stable equilibrium $y_1^* = 0.76$

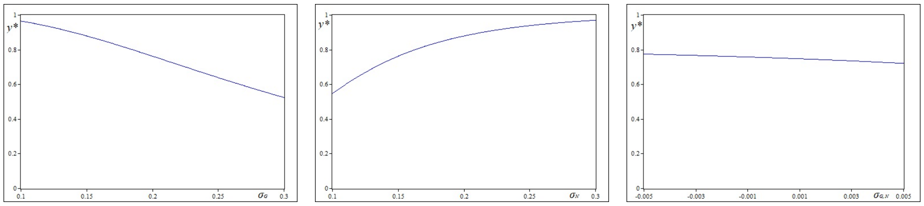


Parameters: $a_G = 1.22, a_N = 1.16, b_G = 0.4, b_N = 0.35, \eta = 0.6, \gamma = 50, \alpha = 2300, \beta = 4000, BT = 140000, \sigma_G = 0.2, \sigma_N = 0.15, \sigma_{G,N} = -0.002, p = 1, r_F = 0.02$. The *i.c.* is 0.2.

Figure 7 presents the bifurcation diagrams for the standard deviations (i.e.

proxy of the financial risk) of green bonds (Fig. 7a), non-green bonds (Fig. 7b), and the covariance between the two typology of bonds (Fig. 7c)[22]. An increase of average financial risk of green bonds σ_G translates into a lower share of these assets in the CB portfolio, and it leads to a rise in the cost of borrowing for these firms. Consequently, the share of green investment gradually falls at the equilibrium (Fig. 7a). The opposite holds for an increase of average financial risk of non-green bonds σ_N as shown in Fig. 7b. The share of green investment rises and the share of non-green investment falls. Lastly, increasing the covariance $\sigma_{G,N}$ from a negative value (correlation) to a positive (correlation) mildly decreases the share of green investment at the equilibrium (Fig. 7c).

Figure 7: Bifurcation diagrams for variances



(a) Panel A: bif. diag. σ_G (b) Panel B: bif. diag. σ_N (c) Panel C: bif. diag. $\sigma_{G,N}$

Parameters: same parameters and *i.c.* of Figure 6.

Note that Figure 6 is obtained given a low speed of technology adoption in the industry: $\gamma = 50$. Starting with the same parameter values and initial condition, Figure 8 demonstrates that an increase in the speed of technology adoption ($\gamma = 200$) causes systemic instability. The firm population periodically shifts between $y = 0.69$ and $y = 0.82$ as shown in Figure 8a. and the phase plot[23] of Figure 8b. In economic terms, the population of firms adopts a technology more rapidly than in the earlier scenario, which creates a periodic adaptation of the other technology as firms choose another technology in each period[24].

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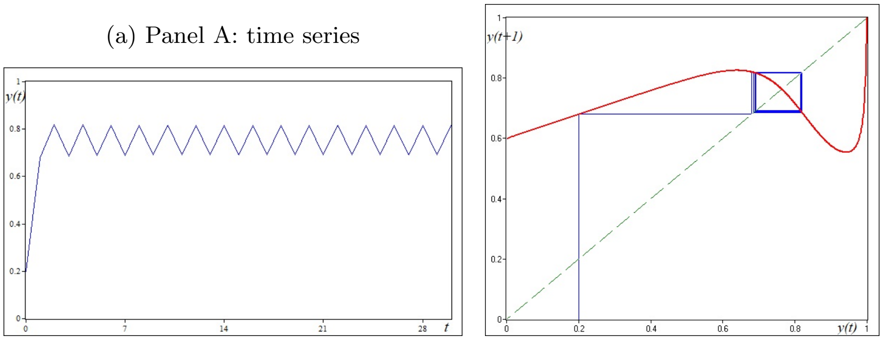
[22] The range of variation of the parameters in this Figure and in all the subsequent bifurcation diagrams is subject to conditions in eqs. (4), (9), (10), (13).

[23] The phase plot shows that the point where the system (in red) intercepts the bisector is the same. However, the increase of γ warps R, lower the point derivative at the previous equilibrium to less than -1. The system undergoes a flip bifurcation.

[24] This is caused by a periodic shift in the profits associated with each technology. While not shown here, we demonstrate this for the following scenario.

Figure 8: Convergence towards a cycle-2 period

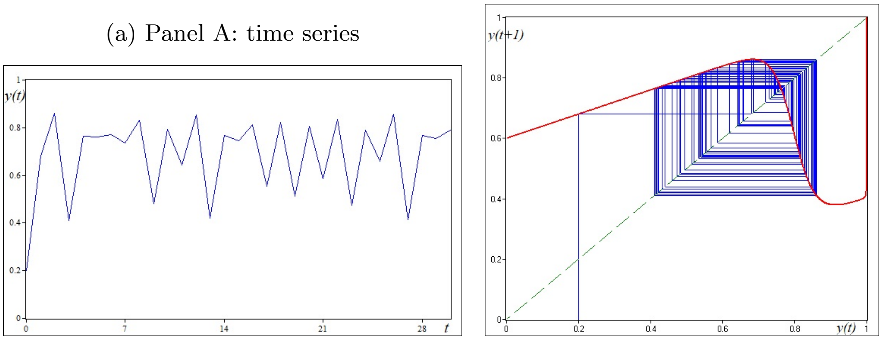

(a) Panel A: time series

(b) Panel B: phase plot

Parameters: same parameters and *i.c.* of Figure 6, except for $\gamma = 200$.

Further increasing γ to 400 leads to the creation of a region of deterministic chaos[25] (Fig. 9). In this specific case, the time evolution of the green investment share is erratic (Fig. 9a). The economic consequence of such erratic motion is a low level of predictability regarding the manufacturing technological adopted in the industry.

Figure 9: Convergence towards a deterministic chaos region


(a) Panel A: time series

(b) Panel B: phase plot

Parameters: same parameters and *i.c.* of Figure 6, except for $\gamma = 400$.

The population dynamics should be similarly affected by the degree of market competition. Figure 10a plots the bifurcation diagram for various values of

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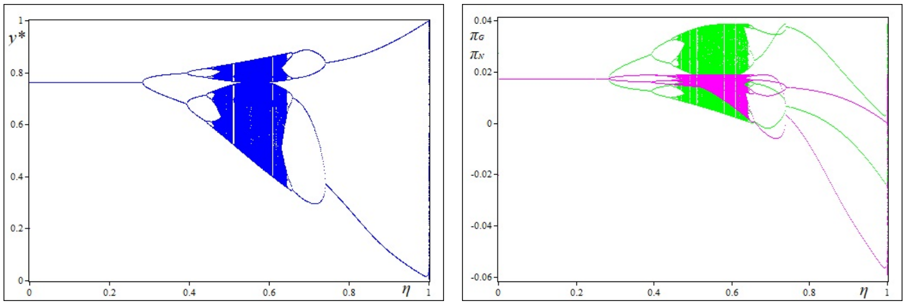
[25] The chaotic attractor characterizes a system that is sensitive dependent on initial conditions (see e.g. Devaney 1986, Lorenz 1989, Medio and Lines 2001).

η. For low values of η, and thus low market competition, the firm population converges to a single interior equilibrium. Bifurcations occur at higher values eventually leading to chaotic behavior for values of η exceeding 0.5. Interestingly, in highly competitive markets we observe non-chaotic but periodic behavior, which is defined by a periodic shift between two equilibria. Consequently, investment in green technology is only chaotic in imperfectly competitive markets. Figure 10b shows the corresponding average profits for both technology. We can see that the periodic shifts and the chaotic behavior at higher η are caused by initially periodic and then chaotic shifts in the firm profits associated with each technology. At very high levels of competition, profits periodically shift between two values for each technology, rendering green investment more profitable in the current period and non-green investment more profitable in the next.

Figure 10: Bifurcation diagrams for η



(a) Panel A: bif. diagram η

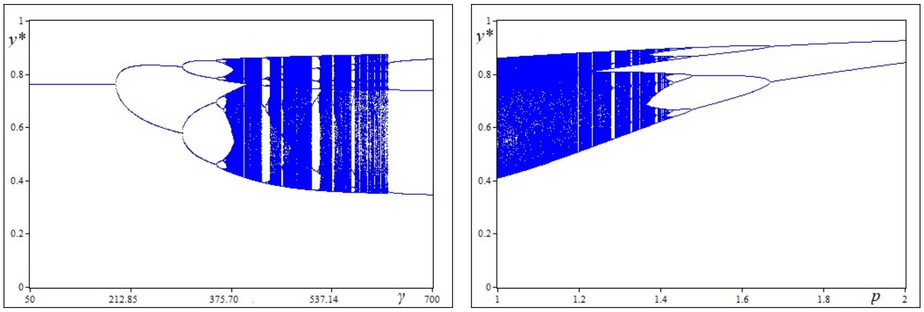
(b) Panel B: bif. diagram of profits for η

Parameters: same parameters and *i.c.* of Figure 6, except for $\gamma = 400$. In panel B, the green curve represents green profit π_G, the pink curve non-green profit π_N.

Figure 11a shows the bifurcation diagram for different rates of technology adoption γ. Here, we observe an effect similar to higher levels of competition. The system bifurcates as adoption rates increase, eventually leading to chaotic behavior at $\gamma = 400$ as demonstrated in Fig. 9. Very high rates of technology adoption eventually also lead to periodic behavior, but here the firm population periodically shifts between three equilibria.

Similar to the previous scenario, a green monetary policy can stabilize investment decisions. Figure 10b shows the impact of p given the neutral monetary scenario in Figure 9. Values of p exceeding 1.4 stabilize technology adoption and eventually lead to periodic shifts.

Figure 11: Bifurcation diagrams for relevant parameters



(a) Panel A: bif. diagram γ

(b) Panel B: bif. diagram p

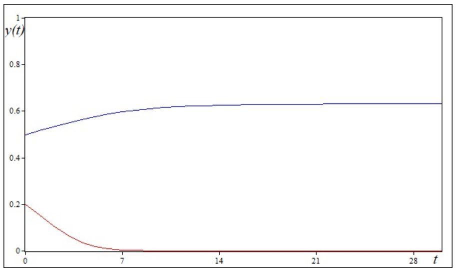
Parameters: same parameters and *i.c.* of Figure 6, except for $\gamma = 400$.

4.4 Two internal equilibria (unstable and stable)

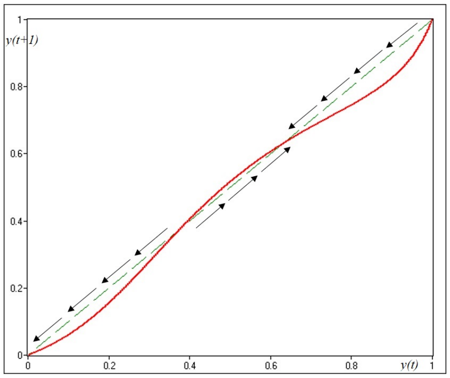
Figure 12 shows the case of two internal equilibria: the first $y_1^* = 0.377$ is unstable ($R'(y_1^*) = 1.25$), the second $y_2^* = 0.631$ is stable ($R'(y_2^*) = 0.75$), and correspondingly equilibrium $y^* = 0$ is stable ($R'(0) = 0.47$) and $y^* = 1$ is unstable ($R'(1) = 3.08$). Figure 12a shows two time series: the red starts from $i.c. = 0.2$ and converges quite rapidly to the equilibrium of full non-green investment $y^* = 0$, whereas the blue starts from $i.c. = 0.5$ and converges after a relative longer period of time to the mixed (or internal) stable equilibrium $y_2^* = 0.63$ where 63% of the firms in the industry employ green technology. The corresponding phase plot is given in Figure 12b, showing the path dependency of the system. A critical share of at least 37.7% of firms adoption a green technology is needed to converge to the upper equilibrium. Any initial condition with fewer firms will remain trapped at the lower equilibrium at which no firm adopts a green technology. Figures 12c and 12d illustrate the firm profits if the population converges to the low or high stable equilibrium, respectively. The low equilibrium is Pareto inefficient and constitutes a *technology trap*.

Figure 12: Scenario of two stable equilibria $y^* = 0$, $y_2^* = 0.63$

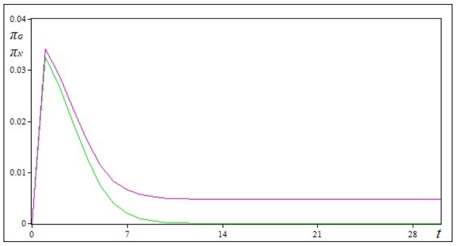
(a) Panel A: time series



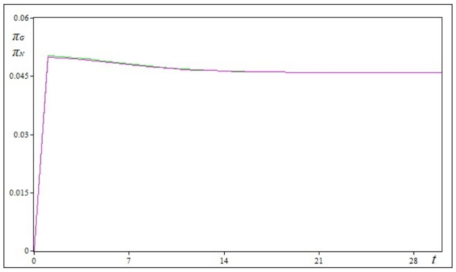
(b) Panel B: phase plot



(c) Panel C: profits low equilibrium



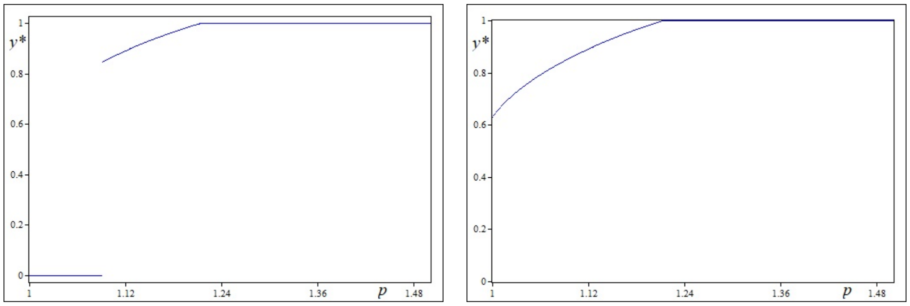
(d) Panel D: profits high equilibrium



Parameters: $a_G = 1.25, a_N = 1.24, b_G = 0.42, b_N = 0.38, \eta = 0.7, \gamma = 300, \alpha = 2300, \beta = 4000, BT = 140000, \sigma_G = 0.2, \sigma_N = 0.15, \sigma_{G.N} = -0.002, p = 1, r_F = 0.02$. In panel A, for the red time series the initial condition (*i.c.*) is 0.2, for the blue time series *i.c.* = 0.5. In panel C *i.c.* = 0.2, in panel D *i.c.* = 0.5. The green curve represents green profit π_G, the pink curve non-green profit π_N.

A green policy by the CB can then help escape this trap as highlighted in Figure 13. The bifurcation diagram of Fig. 13a corresponds to the case of the red time series in Fig. 12a. Indeed, for a *neutral monetary policy* ($p = 1$) the equilibrium value is $y^* = 0$. A *green monetary policy* that progressively augments p causes the firm population to escape the trap. At $p \approx 1.10$, the population shifts from the low to the high equilibrium. To a lesser extent, the beneficial effect can also be observed if the firm population has a critical number of firms, which initially adopt a green technology. However, increasing p does not lead to a shift between the equilibria, but a higher equilibrium value of the higher fixed point. Fig. 13b shows the the situation for an initial condition of 0.5, where the industry is already on the socially optimal equilibrium. Here, moving to a *green monetary policy* ($1 < p < 1.21$) increases the initial mixed equilibrium value from $y_2^* = 0.63$ to $y^* = 1$ for $p > 1.21$.

Figure 13: Bifurcation diagram for p



(a) Panel A: $i.c. = 0.2$

(b) Panel B: $i.c. = 0.5$

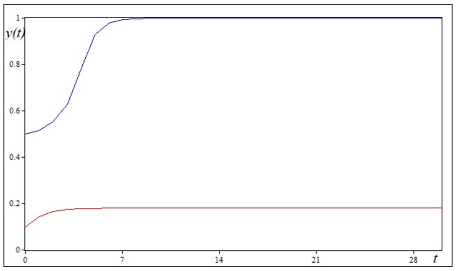
Parameters: same parameters of Figure 12.

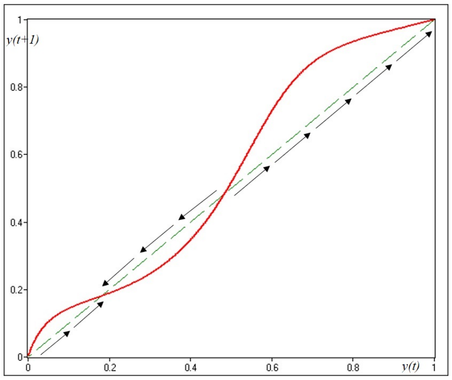
4.5 Two internal equilibria (stable and unstable)

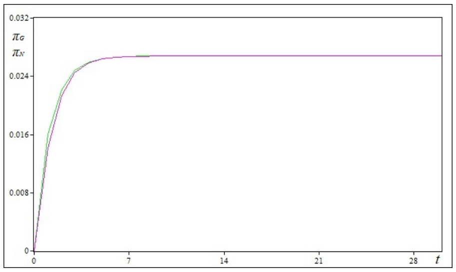
The last relevant scenario is characterized again by two internal equilibria, but with opposite stability properties: the lower interior equilibrium $y_1^* = 0.181$ is stable ($R'(y_1^*) = 0.44$), the second interior equilibrium $y_2^* = 0.480$ is unstable ($R'(y_2^*) = 1.95$), while $y^* = 0$ is unstable ($R'(0) = 3.67$) and $y^* = 1$ is stable ($R'(1) = 0.30$). The scenario is depicted in Figure 14.

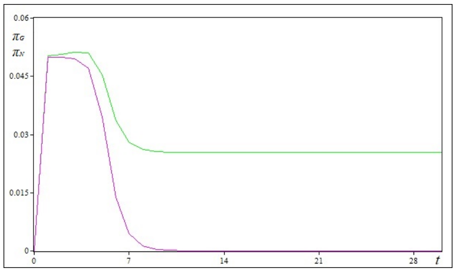
In Figure 14a the red time series starts from $i.c. = 0.1$ and converges quite rapidly to the lower mixed equilibrium, whereas the blue starts from $i.c. = 0.5$ and approaches, after a relatively long period of time, to the equilibrium of full green investment $y^* = 1$. The internal unstable equilibrium $y_2^* = 0.48$, the threshold between the two basins of attractions, is shown in Figure 14b. As previously stressed, the path dependence phenomenon can be better visualized from the phase plot, where for all $i.c. < y_2^*$ the mixed eq. $y_1^* = 0.18$ is reached, while for all $i.c. > y_2^*$ the pure eq. $y^* = 1$ is attained in the long run. The possibility of having two fixed points depending on the initial state of the industry translates into different profit evolution. In Fig. 14c, the firm population converges to the lower mixed equilibrium. Profits for both technologies are equal at 0.027. In Fig. 14d, the population eventually only adopts green technology. Green profits π_G converge to the same profit at 0.027. In this particular scenario, no Pareto inefficient allocation occurs and the green monetary policy of the CB is ineffective.

Figure 14: Scenario of two stable equilibria $y^* = 0.18$, $y^* = 1$


(a) Panel A: time series


(b) Panel B: phase plot

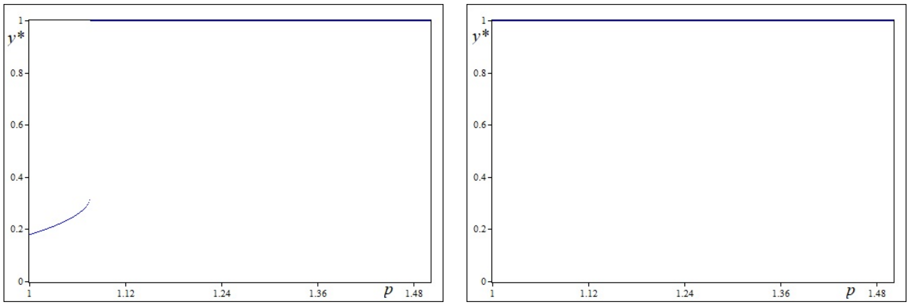

(c) Panel C: profits low equilibrium


(d) Panel D: profits high equilibrium

Parameters: $a_G = 1.22, a_N = 1.25, b_G = 0.3, b_N = 0.42, \eta = 0.7, \gamma = 300, \alpha = 2300, \beta = 4000, BT = 140000, \sigma_G = 0.2, \sigma_N = 0.15, \sigma_{G,N} = -0.002, p = 1, r_F = 0.02$. In panel A, for the red time series the initial condition (*i.c.*) is 0.1, for the blue time series *i.c.* = 0.5. In panel C *i.c.* = 0.1, in panel D *i.c.* = 0.5. The green curve represents green profit π_G, the pink curve non-green profit π_N.

In this scenario, *green monetary policy* CSPP is still useful to encourage the adoption of green technology. The bifurcation diagrams in Figure 15 demonstrate the impact of p in both scenarios, respectively. While the policy is ineffective in the high equilibrium scenario (Figure 15b), increasing p beyond 1.10 helps the firm population to move from the low equilibrium to the high equilibrium (Figure 15a).

Figure 15: Bifurcation diagram for p



(a) Panel A: $i.c. = 0.1$

(b) Panel B: $i.c. = 0.5$

Parameters: same parameters of Figure 14.

5 Conclusion

In recent years, it has become increasingly evident that climate change is one of the main sources of structural change impacting the financial system. Indeed, it affects all agents in the economy, in all sectors and geographic areas with potentially nonlinear dynamics. Moreover, while the quantification of impact, time horizon, and the future pathway is uncertain, there is a high degree of certainty that some combination of physical and transitional risks will materialize in the near future, affecting negatively the stability of the financial systems, and of the economic systems as a whole. Therefore, CB monetary policies have been starting to consider risks related to climate change with the aim to strengthen the role of the financial system to manage risk and mobilize capital for green and low-carbon investments in the broader context of environmentally sustainable development.

In this paper, we developed a model of CSPP that internalized climate-related externalities by means of a tilting factor of the environmental footprint of green and non-green firms. We showed that a shift in the CB portfolio allocation toward bonds issued by low-carbon companies can favor green firms in the market. We modeled firm investment choices with exponential replicator dynamics and explored numerically the dynamical proprieties of the system.

We obtained some main findings. First, some scenarios are characterized by a strong path dependency in which if a large share of firms employed non-green technology, no investment in green technology occurs in the long run, even if the non-green investment equilibrium is inefficient. We define this equilibrium *technology trap* and show that CSPP monetary policy helps the industry leave the technology trap. Second, green and non-green bond riskiness is a key factor that impacts borrowing costs. The larger the average financial risk of bonds, the lower the share of bonds in the CB portfolio, and the larger the firms' borrowing cost. Third, the degree of market competition and of market (im)perfections contribute to amplifying the effects of the green monetary policy by affecting the transmission channel. In the presence of imperfect competition and (or) high degree of market imperfections the *technology trap* is more likely to happen, the green monetary policy seems to foster the adoption of the green technology and to stabilize investment decisions.

Our future research agenda aims at studying two possible extensions. Firstly, we plan to study a model that incorporates the risk of green-washing. A second extension takes into account the interaction between the green monetary CSPP and fiscal policies.

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