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# "SYNCHRONIZATION OF GLOBALIZED ECONOMIES"


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# Synchronization of globalized economies*

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**Abstract**

Does the synchronization of globalized-oscillating economies matter for macroeconomic outcomes? If so, how oscillating economies are synchornized and conform stable networks. In this paper we apply phase oscillator models such as Kuramoto's model to understand synchronization phenomena in networks of countries. Our aim is to study a network of interacting phase oscillating economies, and an adaptation mechanism for the coupling that promotes the connection strengths between those elements that are dynamically correlated. Under these circumstances, the dynamical organization of the oscillators/economies shapes the topology of the graph in such a way that modularity and assortativity features emerge spontaneously and simultaneously. Our results show the conformation of the network and the global and local synchronization measures for the 42 oscillating economies during the period from 1960 to 2018, using Trade (% of GDP) data. Moreover, we obtain the measure of local assortativity in the formation of those economies that more or less interact or are connected. We conclude that the Kuramoto model with networks is a useful tool to study economics synchronization.

**Keywords**: Business Cycles, Complex Systems, Economics of Globalization, Network Topology, Synchronization, Trade Integration.

**JEL Codes**: D85; F14; F15; F41; F44

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# 1 Introduction

Synchronization of oscillating elements is the name that receives the stable behavior in the time in which, due to the coupling between such elements, the dynamics of all of them is the same. In complex networks, different research works have sought the necessary conditions for the appearance of synchronization in the structure of the local scale or in the macroscopic properties. However, it has not been possible to adequately describe the most important characteristics of complex systems, since they frequently exhibit an organization in “modules”, that is, they are composed of subgraphs connected to each other with different internal and external connectivity that form communities. This organization is a limitation in which the local structure can greatly affect the dynamics, regardless of whether homogeneous or heterogeneous networks are treated. The analysis of synchronization processes has benefited by the understanding of the topology of complex networks, and this has contributed to the understanding of their emerging properties (Pikovsky et al., 2003; Boccaletti et al., 2006; Arenas et al., 2008).

To achieve synchronization, several factors must be combined in a subtle equilibrium, the most prominent being the interactions between the elements of the group (for example the interactions of economic agents) and those of each oscillating economy with the environment in which it is located, i.e. in a globalized economic environment (Hyeon-Seung Huh, 2020). So in this work, our interest is to study how this stable organization arises. In addition to studying the appearance of modules in complex socio-economic systems, with the applied model, the dynamics of synchronization, global and local, for oscillating economies are studied through a modified version of the *Kuramoto model* (Kuramoto, 1984, 1975). This research paper is based on a model developed by Avalos-Gaytán et al. (2012); Avalos-Gaytán et al. (2018), in the context of complex networks. Our aim is applied to oscillating economies, in which the formation of the network is dynamically driven by international trade (% of GDP). More precisely, it is considered a network formed by oscillating economies of Kuramoto coupled through links that define the degrees of globalization and evolve according to a dynamic dependent on the synchronization state of the economic system. The links are defined with two states, 1) the links that join nodes/economies of similar dynamics are reinforced, and 2) the links that link nodes/economies with different (asynchronous) dynamics are weakened.[1] In this way,

[1]Given a network, it is defined a partition of its nodes into two or more groups, called modules or communities. Modularity is a measure of the quality of this partition. When in a network the nodes can be assigned to disjoint subsets in which there are many links between them, but few with the rest, the modularity has a high value. When it is not possible to find such a partition, the modularity is low (Newman, 2006).

it is intended to show that the presence of modularity and synchronization (associated with the existence of collective subtasks) are interrelated in a globalized world. It is shown that the presence of modularity is related to the appearance of collective subtasks, which in turn, are coordinated at local and global level. The main objective is to applied the Kuramoto model to a set of economies with topological training, i.e. globalized economies, in order to obtain an efficient synchronization based on the interrelation between the structural properties of the local and global economies, and the dynamic properties of such a system.

Synchronization may occur naturally, usually because of some external driving force (such as the time of day or month or year). Given that synchronization is also a facet of economic agents' performance, it would seem logical to expect synchronization to occur in economic growth when entities are subject to the similar external forces. However, synchronization does not have any agreed upon definition in economics (-unlike in other disciplines, such as physics), and so correlations are often used to denote synchronization. But correlations can be very misleading in terms of dynamics, and highly correlated series can exhibit completely unsynchronized movements in terms of directional movements over time. In economics, another complication of synchronization concerns the Business Cycle theory (see, for example, Moneta and Rüffer (2006) and Degiannakis et al. (2014). Growth convergence is usually assessed in terms of the distribution of economic growth rates, as measured by the growth in real Gross Domestic Product (GDP) over time, and in particular over the span of the business cycle. In Crowley and Schultz (2011) synchronicity is measured in terms of measures derived from recurrence plot analysis methodology. The complication concerning the business cycle is that indeed these episodes of growth usually are extremely synchronized during the contractionary phase of the business cycle, but during the expansionary phase of the cycle, which usually includes sub-cycles, the cycles in growth showed signs of only "intermittent synchronicity". This "intermittancy" is perhaps due to the way that policy measures filter through the macroeconomy, with other factors sometimes overwhelming any policy initiatives. Therefore, we use a well-known developed measure of dynamic synchronicity based on networked connections (see Avalos-Gaytán et al. (2012); Avalos-Gaytán et al. (2018)), which is particularly suited to economics.

From a theoretical perspective, macroeconomic synchronicity is often related to the question on how does globalization affect macroeconomic co-movements across countries. For instance, Matsuyama et al. (2015) demonstrate that globalization can also change synchronicity of productivity fluctuations across countries. They develop a two-country model in discrete time of endogenous fluctuations of innovation cycles. From an empirical viewpoint, Baxter and Kouparitsas (2005); Clark and Van Wincoop (2001); Frankel and Rose (1998); Kose and Yi (2006) develop empirical studies showing that pairs of countries with stronger trade integration have a higher correlation in output growth, i.e. they are higher synchronized.[2] One explanation for this finding is that shocks in one economy alter the demand for foreign intermediate products, since intermediate goods constitute 75 percent of worldwide trade. For example, a positive productivity shock in one country generates an increase in domestic output and income and, hence, an increase in the demand for foreign intermediate goods from its trading partner. The increase in the demand for imports increases the output of the country's trading partner, so the business cycles of the two countries become synchronized. Empirically, this synchronization is stronger for pairs of countries that trade more with each other. Economic theory provides only nuanced guidance about the impact of greater international linkages on output co-movement across countries. On the one hand, theory suggests that si higher bilateral trade between country pairs is associated with more correlated BC. On the other hand, it suggests that if trade increases specialization and if industry-specific shocks are dominant, the degree of BC co-movement should decrease as trade linkages strengthen. Baxter and Kouparitsas (2003, 2005) and Fonseca et al. (2010) provide empirical evidence on the strong and positive effect of trade intensity on business cycle synchronization. This effect may interact with other factors such as product diversification or the convergence in macroeconomic policies according to Inklaar et al. (2008). Moreover, Abbott et al. (2008) show the dependence of business cycle with trade intensity, which is both timely and geographically interrelated, being negative in some cases. These findings have been interpreted as evidence that trade integration leads to business cycle synchronization. However, from a theoretical perspective the standard international real business cycle (IRBC) model, based on Backus et al. (1994), has difficulties in replicating this empirical fact (see Kose and Yi (2001, 2006)). In the latter paper, the authors' baseline model explains only one-tenth of the responsiveness of comovement to trade intensity. This has given rise to the so-called trade-comovement puzzle: Standard models are unable to generate high output correlations arising from high bilateral trade intensity. The standard international business cycle framework cannot replicate this finding, uncovering the trade-comovement puzzle.

Inspired on these ideas, we propose a dynamic synchronization model for oscillating economies in a

[2]Substantial evidence suggests that countries with stronger trade linkages have more synchronized business cycles. Frankel and Rose (1998); Clark and Van Wincoop (2001); Calderon et al. (2007); Baxter and Kouparitsas (2003); Imbs (2004), among others, show that pairs of countries that trade with each other exhibit a high degree of business cycle comovement.

globalized system. In the present paper, we propose that economies are akin to the dynamics of the collective behavior of weakly coupled non-linear oscillators. In the present work, we apply the Kuramoto model with phase lag to the nonlinear dynamics on a graph. We rewrite the original Kuramoto model for a complex network corresponding to undirected graphs.[3] Much effort has gone into understanding the role of the coupling strength (Hong et al. (2002); Arenas et al. (2008); Dörfler et al. (2013)) in the synchronization behavior of small-world and scale-free graphs. Here, we leave the coupling strength term a constant. To put our contribution in a broader context, we used a model of synchronization applied to coupled oscillating economies. [4]

The plan of the paper is as follows. Section 2 formulates the modified Kuramoto model aiming to give an application in economics. Section 3 applies such a model to a set of globalized economies, showing the results on the global, local synchronization force and level of assortativity, considering certain coupling parameters and threshold values for trade relations. Section 4 concludes.

## 2 Complex networks and sychronization

Boccaletti et al. (2006) characterizes complex networks as irregular structures dynamically evolving in time. Application of complex networks goes from transportation networks, phone call networks, the Internet and the World Wide Web, but also systems of interest in biology and medicine, as neural networks or genetic, metabolic and protein networks. The authors also pointed out relevant papers and books dedicated to complex networks (Boccaletti et al., 2006; Arenas and Díaz-Guilera, 2007; Arenas et al., 2008). An economic system studied as a complex network is a recent area of research (see Arenas et al. (2008); Commendatore et al. (2018)). Complex network theory applied to economics deals with the structure and dynamics of an economy forming a network and its dynamic properties over such networks (see Newman (2010); Jackson (2008)). Applications of complex network theory to economics can be found on topics such as supply chain, credit (Battiston et al., 2007), trade, as well as empirical studies on world trade or ownership control among corporations (Vitali et al., 2011), or on buyers-suppliers networks such as the interfirm payment network in Estonia (de la Torre et al., 2016).

[3] In this regard, it is worth mentioning that synchronization of Kuramoto oscillators in directed networks has been subjected to a detailed study (Restrepo et al., 2006).

[4] The subject of coupled oscillators is concerned with the effects of combining two or more systems that generate self-sustained oscillations, in particular, how they mutually affect their rhythms. It is a major topic in natural science, ranging from physics to chemistry to biology to engineering, with thousands of applications.

To understand the structure of the beginning of research on complex networks focused on the study of measures that characterize the topology of real networks. Then the scientific interest go to the development of new mathematical models that would allow them to reproduce a real network. The structure of a real network is the result of the continuous evolution of the forces that formed it, and certainly affects the function of the system (Boccaletti et al., 2006). As a consequence the study on such models became important to analyze the dynamical behavior of large assemblies of dynamical systems interacting via complex topologies, pointing out to the crucial role played by the network topology in determining the emergence of collective dynamical behavior, such as synchronization (Arenas et al., 2006; Gómez-Gardeñes et al., 2007; Boccaletti et al., 2007).

Arenas et al. (2008) defines synchronization as an emerging phenomenon of a population of dynamically interacting units, present in different context such as biology, ecology, climatology, between others. The effect of synchrony has been described in experiments with people communication where the purpose of the common wave length or rhythm is to strengthen the group bond. Synchronization involves, at least, two elements in interaction, and the behavior of a few interacting oscillators has been intensively studied in physics and mathematics literature.

We are interested on the phenomenon of synchronization in complex networks which is more challenging and requires a different approach to be solved. We apply the Kuramoto model (Kuramoto, 1984; Acebrón et al., 2005a) because it is the first model to understand the phenomena of synchronization, also is the appropriate model for coupled oscillators by applying the sine function to their phase differences. In addition, this model is very flexible and can be adapted to different contexts and shows great diversity of patterns for synchronization (Arenas and Díaz-Guilera, 2007).

## 2.1 The model

Kuramoto (1984, 1975) worked out a mathematically tractable model to describe the synchronization phenomenon, specifically models the behavior of many coupled oscillators. In the most common form of the model, each oscillator is considered to have its own natural frequency ($\Omega_i$), and is also coupled to the other oscillators. Given $N$ oscillators, the phase of oscillator $i$ denoted by $\theta_i$ evolves in time according to:

$$\dot{\theta}_i = \Omega_i + \frac{\sigma}{N} \sum_{j=1}^{N} \sin(\theta_j - \theta_i), \quad i = 1, \ldots, N,$$

where $\Omega_i$ stands for the natural frequency of oscillator $i$, $\sigma$ is the coupling constant, and the factor $1/N$ is incorporated to ensure a good behavior of the model in the thermodynamic limit $N \to \infty$. The frequencies $\Omega_i$ are distributed according to a given probability density $\Omega \mapsto g(\Omega)$, that is usually assumed to be unimodal and symmetric about its mean frequency. $\dot{\theta}_i$ describes the dynamics of each oscillator in time (Arenas et al., 2008).

In this work we use an adaptation of the Kuramoto equation for complex networks (Avalos-Gaytán et al., 2012; Avalos-Gaytán et al., 2018), that is:

$$\dot{\theta}_i = \Omega_i + \underbrace{\frac{\sigma}{N}\sum_{j=1}^{N} w_{ij}\sin(\theta_j - \theta_i)}_{(1.1)}, \tag{1}$$

where $w_{ij}$ are the elements of the connectivity matrix $W$, which represent the topology of the complex system. In Eq. (1) the term (1.1) is a diffusive coupling that express a phase difference between oscillators, in which $(\sigma/N) \times w_{ij}$ is the coupling intensity. We are using a function that evolves in time the connectivity between all pairs or oscillators, i.e.:

$$\dot{w}_{ij} \quad = \quad (p_{ij} - p_c)w_{ij}(1 - w_{ij}) \tag{2}$$

where $w_{ij}$ is the weight of the link between each pair of oscillators $i$ and $j$. Note that $w_{ij}(1 - w_{ij})$ is the driving force for the weight dynamics with two attractors, leading each weight to asymptotically converge to either one of the values in $[0, 1]$. The parameter $p_c$ is a threshold correlation which determines whether a weight is reinforced ($p_{ij} > p_c$) or weakened ($p_{ij} < p_c$). The parameter $p_{ij}$ is used to measure the correlation between pairs of oscillators, it considers the average of the phases and is defined as:

$$p_{ij} \quad = \quad \left|\cos\left(\frac{\theta_i - \theta_j}{2}\right)\right| \tag{3}$$

when $p_{ij} \to 1$ the two oscillators are synchronized ($\theta_i = \theta_j$), when $p_{ij} \to 0$ there is no synchronization.

To analyze the synchronization phenomenon, we have applied the global and local synchronization measures. Global synchronization is measured with the global order parameter proposed by Kuramoto, this parameter is applied since it has shown to give good results to measure the level of global synchronization

(Kuramoto, 1984; Boccaletti et al., 2002, 2006; Arenas et al., 2006; Strogatz and Mirollo, 1988; Acebrón et al., 2005b), it is defined as:

$$R^G = \left\langle \frac{1}{N} \left| \sum_{j=1}^{N} e^{\mathrm{i}\theta_j(t)} \right| \right\rangle_t ,$$

in which $\langle \cdot \rangle$ is the average over time $t$.

To quantify the level of local synchronization we measure the degree of similarity of the dynamics of a node with its neighbors at each instant of time, defined as follows (Avalos-Gaytán et al., 2012):

$$R_i^L = \left\langle \left| \frac{\sum_{j=1}^{N} w_{ij} e^{\mathrm{i}\theta_j(t)}}{\sum_{j=1}^{N} w_{ij}} \right| \right\rangle_t ,$$

hence, the average level of local synchronization is:

$$R^L = \frac{1}{N} \sum_{i=1}^{N} R_i^L .$$

By this, a system can exhibit a low global synchronization level, but have a high local synchronization level.

An important structural property to characterize the topology of a system is assortativity, according to Newman (2002), this is the preference that elements of a system have for joining with others that are similar to them. This measure can be very variable, but those who are dedicated to the study of complex systems frequently study assortativity based on the degree of the system elements. Characterizing a system with this measure is useful to approximate complex system models, to the behavior of real systems. This measure is equivalent to Pearson's correlation coefficient, in a system it is measured between the degree of an element and the average degree of its neighbors.

If this coefficient is close to one, it implies that the elements of the system with a high degree tend to be connected with other elements with a high degree. Otherwise, when this coefficient is close to minus one, it implies that high grade elements tend to connect with low grade elements. Values close to zero of this coefficient imply that they are connected independently of the degree they have. Here we measure the assortativity defined by Newman (2002, 2003) as follows:

$$\mathcal{A}^L = \frac{1}{\sigma_q^2} \sum_{k=1}^{N-1} \sum_{l=1}^{N-1} kl(e_{kl} - q_k q_l),$$

where $k$ and $l$ are respectively, the grade of the element $k$ and $l$, $q_l$ is the distribution of the residual degree, $e_{kl}$ is the joint probability distribution of the residual degrees between pairs of elements of the system, the degrees of the elements without counting their union; Finally, $\sigma_q^2$ is the variance associated with the distribution of $q_k$ for $k = 1, \ldots, N-1$. For a better understanding of this measure and details we recommend reviewing Newman (2002); Girvan and Newman (2002).

Notice that equation (1), equation (2), and equation (3) represent our system of dynamic equations, which cannot be considered alone or separately. Therefore we may state the following remarks.

**Remark 1.** *We can determine the fixed and stable points for $\dot{w}_{ij}$ and what happens in extreme cases, when $p_c$ is zero or one. For example, $\dot{w}_{ij}$ is zero when $w_{ij}$ is zero or one. When $\dot{w}_{ij} = 0$ for $w_{ij} = 1$, $w_{ij}$ is a stable fixed point, it doesn't matter if $w_{ij} = 0.9999$ since $\dot{w}_{ij}$ will tend to move $w_{ij}$ to the value one, so $w_{ij}$ is a stable fixed point.*

**Remark 2.** *When $\dot{w}_{inj} = 0$ for $w_{ij} = 0$, $w_{ij}$ is an unstable fixed point but with the difference that if, for example, $w_{ij} = 0.001$, so $\dot{w}_{ij}$ makes $w_{ij}$ more than zero, so $w_{ij} = 0$ is an unstable fixed point. Only if $w_{ij}$ is zero with infinite precision, $w_{ij}$ remains zero; but if not, it is enough that $w_{ij}$ be $0.00000001$ so that $\dot{w}_{ij}$ moves $w_{ij}$ towards the stable fixed point.*

**Remark 3.** *The effect that $p_{ij} - p_c$ has on $\dot{w}_{ij}$ is to change the fixed points, that is, when $p_{ij} - p_c > 0$ it implies that $w_{ij} = 1$ is a stable point and $w_{ij} = 0$ is an unstable point. When $p_{ij} - p_c < 0$ it implies that $w_{ij} = 1$ is an unstable point and $w_{ij} = 0$ is a stable point.*

For extreme cases of $p_c$, it is known that if this parameter is zero, $p_{ij} - p_c$ will always be positive and in this case all the links will go to one, regardless of whether the nodes/economies are synchronized or not, and if so, the network will end up in which the nodes will be all connected to everyone. On the contrary, when $p_c$ is one, it is necessary that $p_{ij} - p_c$ will always be negative and in this case all the links will go to zero, also regardless of whether the nodes/economies are synchronized or not, and the network resulting tends to be totally disconnected.

With this model we must study the different combinations for the parameters $p_c$ (threshold correlation) and $\sigma$ (coupling force) for different initial conditions of the natural frequencies $\Omega_m$ and the weights $w_{mn}$'s. The above with the aim of determining the optimal initial conditions for which the model allows us to show that effectively synchronization in networks of globalized economies, where dynamic and topological properties influence each other and that it requires a modular structure.

## 3 Model setup to globalized economies

Following the above previous considerations, in order to apply Kuramoto model, we consider an economic complex system. That is, economic systems with interconnected spatial structures (globalizaed economies) where different types of decisions and interactions take place, for example the interactions among international or regional trading partners at the macro-level as economic network structures. Within these structures, the spatial distribution of economic activities is evolving through time following complex patterns determined by economic, geographical, institutional and social factors (Commendatore et al., 2018).

Our model applies to a set of 42 economic countries, which from now on will represent $N = 42$ oscillating economies, during the period 1960-2018. Our data is defined by Trade (% of GDP) variable, from the database: World Bank National accounts data, and OECD National Accounts data files, The World Development Indicators. Trade is defined as the sum of exports and imports of goods and services measured as a share of gross domestic product (see `https://data.worldbank.org/indicator/NE.TRD.GNFS.ZS`).

The economies considered in our network are: Algeria, Argentina, Australia, Austria, Bangladesh, Belgium, Bolivia, Botswana, Brazil, Burkina Faso, Canada, Chile, China, Colombia, Costa Rica, Cote d'Ivoire, Denmark, Arab Republic of Egypt, Finland, France, Greece, Hong Kong SAR China, Iceland, India, Ireland, Italy, Japan, Korea Rep., Luxembourg, Malaysia, Mexico, Netherlands, Nicaragua, Nigeria, Norway, Singapore, South Africa, Spain, Turkey, United Kingdom, United States, and Uruguay.

In the complex network the economies represent oscillators, and the connections between them are represented by links weighted by $w_{ij}$. The oscillating economies evolving in time will be modeled through Kuramoto Eq. (1), the connectivity between all pairs or oscillators through Eq. (2) and the parameter $p_{ij}$ used to measure the correlation between pairs of oscillators by Eq.(3).

We compute the panel correlations matrix (ordinary, Pearson product moment) in order to define the connectivity matrix $w_{ij}$. That is, $w_{ij}$ are the elements of the adjacency matrix $A$, which represent the topology of the complex economic systems. More specifically, elements $w_{ij} = 1$ if two nodes/economies $i$ and $j$ are connected, and $w_{ij} = 0$, otherwise. Where the level of connection is defined by the correlation matrix $A$.[5] According to Avalos-Gaytán et al. (2018), the natural frequencies $\Omega_i$ are randomly chosen in

[5] Contemporaneous correlations between macroeconomic variables are often used to examine the nature of relationships

the range $(0.8, 1.2)$ with a uniform distribution probability function. That is, $\Omega_i$ stands for the natural frequency of each oscillating economy $i$, i.e. the initial conditions of each economy. $\sigma$ is the coupling constant acting as a first global parameter and can be interpreted as a globalization measure for the economies, or the level of trade between the economies, when $\sigma \to 1$ there is a lot of interaction between the economies, when $\sigma \to 0$ there is little or no interaction between economies.

The phase of each oscillating economy are defined in $0 < \theta_i < 2\pi$. The $p_{ij}$ correlations are defined in $0 \leq p_{ij} \leq 1$. $p_c \in [0, 1]$ is a threshold value defining the probability of strength trade or exchange or economic relations. In this paper, it is our goal to find such a threshold value that will define the levels of synchronization between economies. Being $p_{ij}$ the probability of correlation between the oscillating economies $i$ and $j$. If $p_{ij} > p_c$ then (the link weight is strengthened) there is a correlation between the $i$ and $j$ economies, otherwise a value $p_{ij}$ less than $p_c$ indicates that the economies lose relationships or do not have a probability of correlate between them.

## 3.1 Computational results

Notice that applying the model presented above in Section 2, we aim to study three important measures which are usually applied to analyze the synchronization phenomenon: global and local synchronization, and local assortativity.

The reported simulations are carried out with complete system of $N = 42$, meaning that the initial topology has $N(N-1)/2 = 861$ interactions between oscillators. The intrinsic frequency $\Omega_i$ of each node is chosen independently at random from a uniform distribution $g(\Omega)$ defined in the interval [0.8, 1.2]. The first parameter that we allow to change in the experiments is the correlation threshold $p_c$, from 0.050 to 0.950 with increments of 0.025. The second parameter is the variation of the coupling constant $\sigma$, we are using the following values 0.2, 0.26, 0.30, 0.35, 0.40 and 0.60.[6]

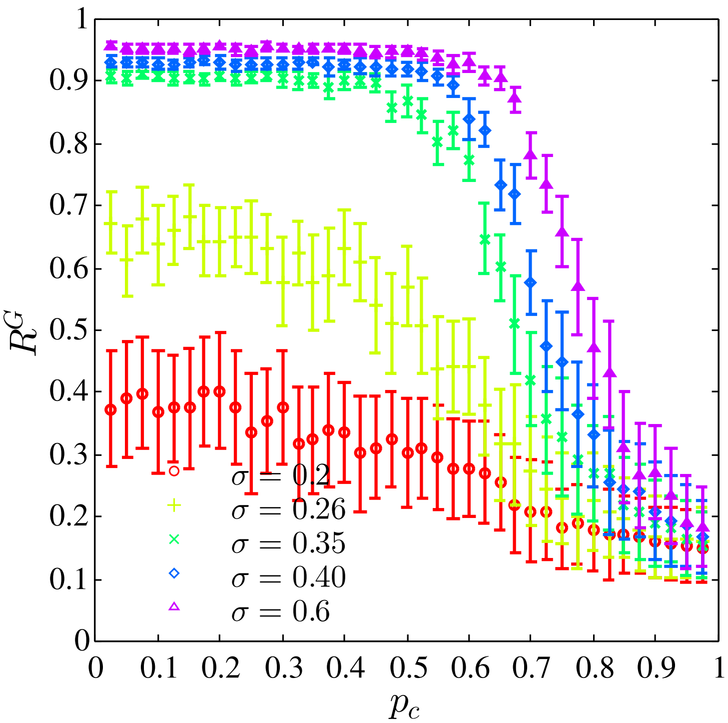
Figures 1 and 2 shows the degree of global ($R^G$) and local ($R^L$) synchronization, respectively, in the final topology of the system evolution under different $p_c$ and $\sigma$ values.

Figure 1 shows the global synchronization, that is to say how a particular economy in the complex network is synchronized with all the rest of the economies.

between different countries (see for example, Obstfeld and Rogoff (2000))

[6]In the simulation of the system of equations it is allowed to evolve over time and 30 repetitions are made, so the results show the average. The experimentation was carried out on a DELL XPS 15 computer, with Intel Core i7 processor, 8GB Memory.

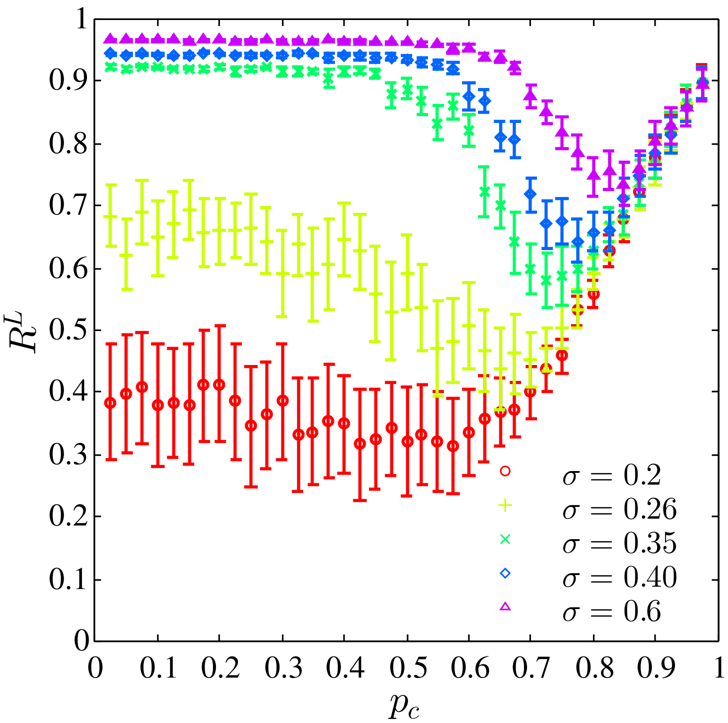
Figure 1: **Global synchronization** quantifies how well you are globally synchronized with all the rest of the network.




The results shown in Figure 1 indicate that there is global synchronization within our network of economies for coupling $\sigma$ values 0.26, 0.30, 0.35, 0.40, 0.60, and threshold correlations of $p_c \in [0.025, 0.70]$. This is represented by the initial or high part of the waterfall that is formed in Figure 1, this initial or high part indicates high levels of global synchronization in our economies studied during the period of time considered. Note that for values of $p_c$ greater than 0.7, global synchronization begins to fall. This behavior is because the barriers to trade are so high, the threshold correlation to trade is so high. However you may have low global synchronization, but good local synchronization, as it looks in the following Figure 2.

Results in Figure 2 show that local synchronization is increasing from $p_c = 0.7$ approximately, for all $\sigma$ values, even when $\sigma = 0.2$. But also even for small $p_c$ values, and that means that in general, we get good local synchronization.

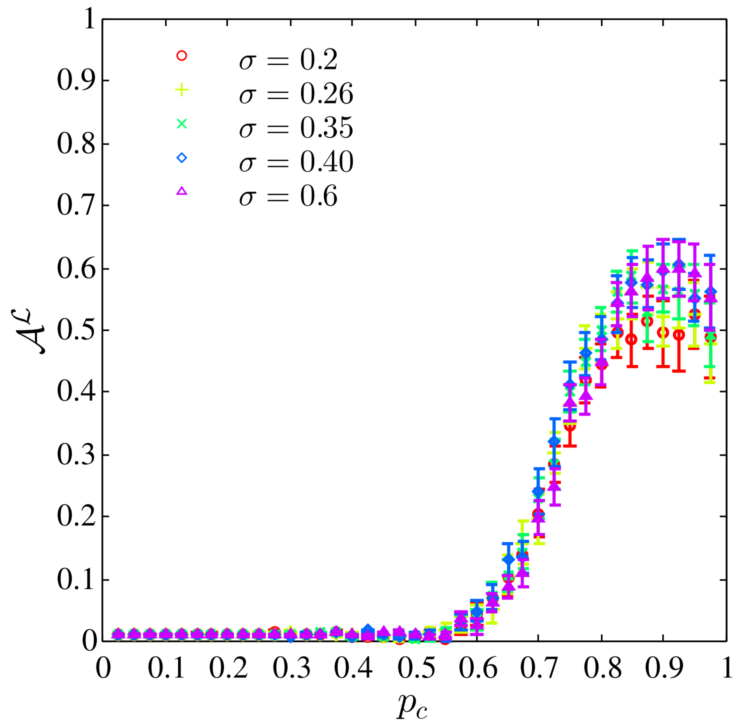
Figure 2: **Local synchronization** measures the degree of similarity of the dynamics of an economy/oscillator with its neighbors.




Recall that local synchronization measures the degree of dynamic similarity of an economy/oscillator with its neighbors. That is to say, as a particular economy is dynamically related to its neighboring economies, and as we can see (Figure 2), we obtain that for high correlation threshold values ($p_c = 0.7$), local synchronization grows, that is, the degree of similarity between neighboring economies is greater, regarding the coupling value $\sigma$.

Finally, let's analyze the local variety of economies, the local assortativity ($\mathcal{A}^{\mathcal{L}}$). Figure 3 show that the assortativity exhibit a rapid transition when $p_c$ is around $0.6 - 0.7$. For smaller $p_c$ values when local and global synchronization take their maximum values, there is a complete lack of clustering, however for larger values when the global synchronization is low, but local is high, the final topology increase the clusterization regardless the $\sigma$ value.

Figure 3: **Local assortativity** measures the degree of similarity between one particular economy/oscillator and the average degree of its neighbors.




From Figure 3, we conclude that the measurement of local assortativity shows high levels of connection for the network of 42 economies given their topological (economic) structure. We obtain that for $p_c$ values ranging from 0 to 0.5, the local assortativity coefficient is zero indicating that these 42 economies are very good connected or have high degrees of interaction, regardless of the economic degrees that are representing. For $p_c$ values larger than 0.7 and for all $\sigma$ values the final topology present good clusterization.

Figure 4 show the final network considering $p_c = 0.7$ and $\sigma = 0.2$. We also prove with $p_c = .7, .8, .9$ and $\sigma = 0.26$ (Figure 5), and $\sigma = 0.60$ (Figure 6) but the final topology is the same.

Figure 4: Network graph for the 42 oscillating economies, final topology to $\sigma = 0.20$ and $p_c = 0.7$

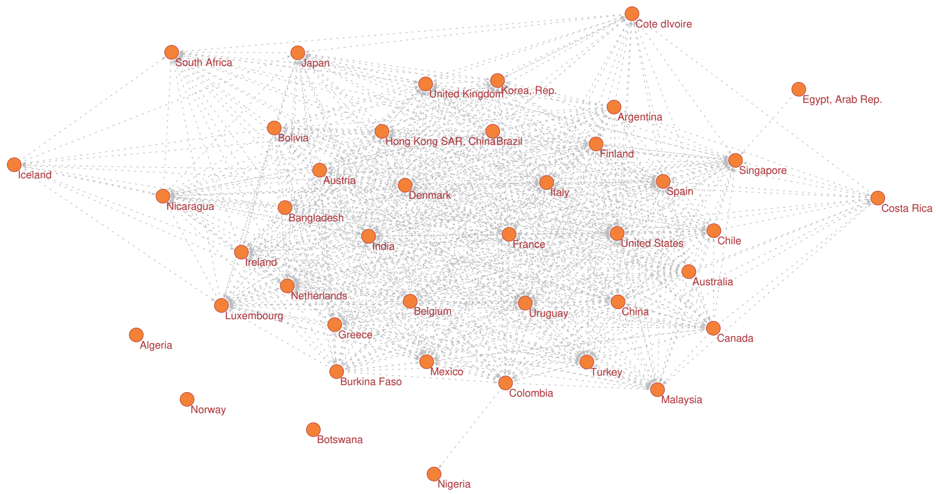
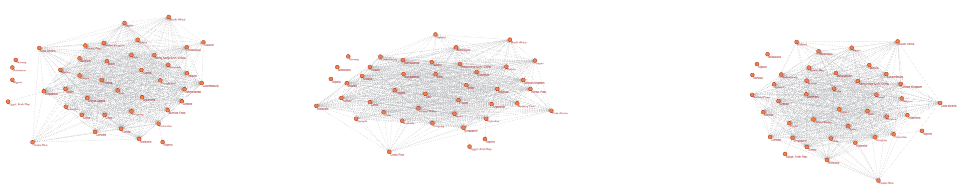
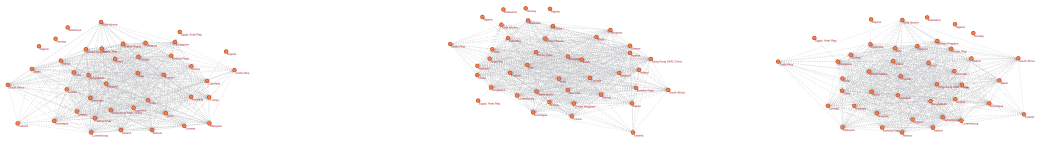



Figure 5: The final topology to $\sigma = 0.26$ and $p_c = 0.7, 0.8, 0.9$.



(a) $p_c = 0.7$ (b) $p_c = 0.8$ (c) $p_c = 0.9$

Figure 6: The final topology to $\sigma = 0.6$ and $p_c = 0.7$, 0.8, 0.9.



(a) $p_c = 0.7$ (b) $p_c = 0.8$ (c) $p_c = 0.9$

Figure 4 shows that economies with lower degree/intensity of interaction (lower number of connections) are: Egypt Arab Republic, Nigeria, Iceland and Costa Rica. In the opposite case, the countries with the highest number of connections are: Argentina, Brazil, Ireland, Netherlands, Spain, United States (all counties with degree equal 34), Austria, Bangladesh, Denmark, Finland, France, Hong Kong SAR, China, India, Italy (all countries with degree equal 35). Algeria, Botswana and Norway in all cases end up leaving the system.

Table 1 shows the complete list of the final degree for each country corresponding to the final topology to $\sigma = 0.20$ and $p_c = 0.7$.

| Country | Algeria | Botswana | Norway | Arab Rep. of Egypt | Nigeria | Iceland | Costa Rica | South Africa | Cote dIvoire | Malaysia | Japan | Nicaragua | Burkina Faso | Canada | Turkey | Mexico | Uruguay | Australia |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Degree | 0 | 0 | 0 | 1 | 1 | 12 | 15 | 22 | 23 | 27 | 28 | 29 | 30 | 30 | 31 | 32 | 32 | 33 |

| Country | Belgium | Bolivia | Chile | China | Colombia | Greece | Korea, Rep. | Luxembourg | Singapore | United Kingdom | Argentina | Brazil | Ireland | Netherlands | Spain | United States | Austria | Bangladesh |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Degree | 33 | 33 | 33 | 33 | 33 | 33 | 33 | 33 | 33 | 33 | 34 | 34 | 34 | 34 | 34 | 34 | 35 | 35 |

| Country | Denmark | Finland | France | Hong Kong SAR | India | Italy |
|---|---|---|---|---|---|---|
| Degree | 35 | 35 | 35 | 35 | 35 | 35 |

Table 1: $N = 42$ oscillating economies with their final degree to $\sigma = 0.20$ and $p_c = 0.7$.

For the network of economies we just analysed and considered their data on trade (% of GDP), we claim that there are three channels through which an increase in bilateral trade may increase business cycle synchronization: (i) Increased bilateral trade resulting in a higher correlation between each country's technology shocks (local synchronization); (ii) increased, in general, trade resulting in a higher correlation between each country's share of expenditure on domestic goods (global synchronization); and (iii) increased trade raising the correlation between the topological economic structure of the economies (local assortativity), for instance raising domestic import penetration ratio and foreign technology shocks.

# 4 Concluding remarks

We apply a synchronization model *à la* Kuramoto where the initial conditions, that is to say the coupling force and the threshold probabilities of correlation between the economies are given, and so it rises a certain degree of global and local synchronization for the oscillating economies.

We show that there may be a drop on global synchronization for a high correlation threshold value ($p_c = 0.7$), because the probability of correlation between any pair of economies in a given network is low than such a threshold, while at the same time high local synchronization also occurs for such threshold values, since one particular economy dynamically behaves so similar to its neighbors. That is to say, globally the synchronization between the economies can fall, but the synchronization that occurs between pairs of neighboring economies is high, and more when there is sufficient coupling between them, although the threshold value of starting trade is high.

On the other hand, local assortativity, or connectivity between global economies is strong when the correlation threshold value is around 0.5. We show the formation of networks between these globalized economies, distinguishing which of these economies are more or less connected to each other.

Thus the application of the Kuramoto approach is a subtle tool for the study of economies as a complex system that seeks synchronization. This gives rise to a great gamma of future research, i.e. in finance, in development economics, in economic history, in environmental economics, and in all those areas of the economy that try to look at how congestion effect arises, synchronization, spills, and/or the formation of interactive or well connected networks. Or even more, to provide empirical evidence in favour of the view that the economic unions (like European/Monetary Union) represent a group of countries that follow synchronously speeds of development in terms of their basic economic features and figures and especially with respect to the fluctuations in their economic welfare, and growth.

**Compliance with Ethical Standards: The authors declare that they have no conflict of interest.**

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