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CORRUPTION, INCOME INEQUALITY, AND TAX
CONTROL”**

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Games and Network Structures on Corruption, Income Inequality, and Tax Control

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Abstract

We study taxpayers' decisions according to their personal income, individual preferences with respect to the audit and tax control information perceived in their social environment. We consider that citizens are classified by two social groups, the rich and the poor. When public authorities are corrupt, we show that the poor group is the most affected by corruption. However, when taxpayers are corrupt or tax evaders, we implement mechanisms to audit and control this corrupt behaviour. We show that this situation can be represented by several well-known theoretical games. Then, evolutionary dynamics of the game in networks considering that each taxpayer receives information from his neighbours about the probability of audit is analyzed. Our simulation analysis shows that the initial and final preferences of taxpayers depend on important parameters, i.e. taxes and fines, audit information and costs.

Keywords: Behavioral economics; Corrupt behavior; Income distribution; Income taxation system; Network Games; Population games.

JEL Codes: C72; C73; O11; O12; O55; K42.

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1 Introduction

The misuse of public office for private gain in a manner that contravenes the rules of the game (so it is defined corruption) has been found responsible for losses in GDP growth (Mauro 1996; Leite and Weideman 1999; Tanzi and Davoodi 2000, Abed and Davoodi 2000), in the ratio of investments to GDP (Mauro 1996; Ades and Di Tella 1997; Tanzi and Davoodi 1997), in the ratio of public education and public health spending to GDP (Mauro 1998), in the ratio of tax revenues to GDP (Ghura 1998), in some measures of government revenues to GDP ratio (Tanzi and Davoodi 2000), and finally in the amount of foreign direct investment (Habib and Zurawicki 2001).

Corruption may also affect income inequality by means of other variables like quality and quantity of public services (especially in the education and health sectors), by reducing the effectiveness of public spending. Inequality of opportunities like having a healthy body and an equal access to a decent education may have reflections on future income perspectives, and therefore on income inequality. Hence, corruption is likely to affect the investment in and formation of human capital through its impact on the effectiveness, outcomes and composition of public spending, which in turn may undermine long term sustainable development, economic growth and equality.

Both theoretically and empirically, it has been shown how corrupt practices on the part of public officials can compromise growth and exacerbate inequalities by distorting incentives, destroying opportunities, squandering resources and perverting public policy. For example, some research papers studying the relationship between corruption and income inequality are, among others, the following:

1. Andres and Ramlogan-Dobson (2011) show that lower corruption is associated with higher inequality in Latin America countries. This result is in contrast with previous studies but the panel of LA countries makes such results robust for several reasons, institutional and cultural aspects.
2. Apergis, Dincer and Payne (2010) investigate the causality between corruption and income inequality within a multivariate framework for the U.S. over the period 1980 to 2004. Using cointegration techniques, they detect a long run relationship between corruption and income inequality and a bidirectional Granger-causality between these two variables.
3. Chong and Gradstein (2007) investigate theoretically and empirically the relationship between inequality and institutional quality, placing its findings (of a double causality between these two variables) in the context of the conflicting evidence as to how corruption affects inequality.
4. Dincer and Gunalp (2005) use a panel dataset for the US states and find robust evidence that an increase in corruption (measured by the number of convictions for crimes related to corruption) increases the Gini coefficient of income inequality and decreases income growth. They justify this fact by saying that the benefits from corruption are likely to flow to better connected individuals and groups who typically belong to higher income groups. Better connected individuals are more likely to get the most profitable government projects, undermining the government's ability to ensure equitable distribution of resources.
5. In Indonesia, Olken (2005) studies the extent of corruption in a large transfer programme distributing subsidised rice to poor households. Using survey data, he finds that losses due to corruption may be large enough to outweigh the redistributive potential of this social welfare programme, because about 18% of the rice disappeared between the time it left government warehouses and the time it reached poor households, and comparing the costs of this corruption with the potential redistributive benefits from the programme, corruption was sufficiently large to outweigh the intended benefits of the programme. This suggests that corruption can seriously hamper the redistributive efforts of the social programmes, so income inequality.

6. De Gregorio and Lee (2002) find that average years of schooling, and other educational factors, contribute positively to a more equal distribution of income. Li et al. (1998) and Barro (2000) are examples of others who also find that more schooling appears to have an income equalizing effect.
7. Gupta and Alonso-Terme (2002) show that high and rising corruption increases income inequality and poverty. They conclude that policies that reduce corruption will most likely reduce income inequality and poverty as well.
8. Gupta et al. (2002), based on a cross-country analysis, find that corruption increases income inequality through lower economic growth, biased tax systems favouring the wealthy and well connected, lower levels and effectiveness of social spending, and unequal access to education and public services.
9. Using panel data from African countries, Gymah-Brempong (2001) claims that a one-point increase in the corruption index is associated with a seven-point increase in the Gini coefficient of income inequality. Despite the availability of a panel dataset, however, this author does not address the issue of causality.

This paper seeks to make a further contribution to the research analysis of corruption, income distribution, and tax control, by applying game theory, and network analysis as a powerful tool to study the behavioral dynamics of corruption in a tax system. The tax system is one of the most important mechanisms of state regulation. A significant part of this system is tax control, which provides receiving taxes and fees in the state budget. Wide class of models, such as Boure and Kumacheva (2010), Chander and Wilde (1998), and Vasin and Morozov (2005), have used a game-theoretical approach, where “the threshold rule” was formulated. This rule defines the value of auditing probability which is critical for the decision of taxpayers to evade taxation or not. However, in real life it is difficult to implement tax inspections with the threshold probability because this process requires large investments from the tax authority, while it has substantially limited budget. Hence, the tax authority needs to find a way to stimulate the population to pay taxes in accordance with their true level of income. Previous studies (see Nekovee et al., 2007; Tembine et al., 2010) have shown that information dissemination has a significant impact on the behavior of agents in various environments, such as the urban population, the social network, labor teams, etc.

Taking into account previous research (Accinelli and Sanchez Carrera, 2012; Antocia et al., 2014; Antunes et al., 2006; Apergis et al., 2010; Bardhan, 1997; Barro, 2000; Kumacheva. et al, 2018; Bloomquist, 2006), this research paper studies the propagation of information about upcoming tax inspections as a tool to stimulate the population to pay taxes honestly. This approach allows tax authority to optimize the collection of taxes within the strong limitation of budget. In this research paper, we assume that the population of taxpayers is heterogeneous in its perception of such an information. Additionally to previous research (Gubar et al., 2015, 2017) susceptibility of each agent depends on its risk-status, due to her natural propensity to risk. As in previous studies we take into account three possible risk-statuses: risk-loving, risk-averse and risk-neutral. These three statuses define the behavior of taxpayers, according to their intentions to evade the tax payment. For example, risk-averse taxpayers prefer to avoid the punishment from the tax authority, therefore, they pay taxes. Risk-loving taxpayers choose risky behavior and try to evade the tax payment. Risk-neutral taxpayers follow to flexible and adaptive behavior, they can behave as a risk-loving or risk-averse taxpayers in different conditions. Economic environment of each individual also impacts on the perceiving of incoming information. However, in this research study we consider taxpayers that have risk-neutral status composing a population of taxpayers. In contrast to many different works, where information transmits during random matches of agents, we consider only structured population and hence information can be transferred only between connected agents.

Social connections of each taxpayer mathematically can be described by using networks structures and its modifications. Here we assume that tax authority injects information about future audits to the population and thereby the initial share of Informed agents is formed. Informed agents can spread

information over their contact network and thereby the ratio between informed and uninformed agents in structured population is changed. Propagation of information also initiates migration of economic agents between two subgroups: those who pay taxes honestly and those who evade payments. However in real-life situations many agents communicate mostly with their family and friends, such kind of interactions can be defined by the network of contacts (see Tembine et al., 2010; Gubar et al., 2015). According to all these reasons we formulate an evolutionary model on the network on the network which describes the variation of taxpayers' behaviour. If a taxpayer switches on another status, then she transfers to the new subgroup, and thus the qualitative structure of the population is changed. This population process resembles an evolutionary game. Therefore, we can use the tools of evolutionary game theory, such as stochastic evolutionary dynamics, to describe the changes in the taxpayers' behaviour. A taxpayer, who receives the opportunity to change her status following a revision protocol, chooses an opponent at random and switches from status i to status j according to the conditional rate (Sandholm, 2010; Gubar et al., 2017; Sanchez Carrera et al., 2018). In other words, the taxpayer can compare her behaviour with the behaviour of the random agent. If the exemplified strategy gives better payoff, then she changes her status (strategy). We estimate the initial and final distribution of taxpayers which prefer to evade taxation in series of numerical simulations.

This paper is structured as follows. Section 2 develops a simple model showing that if all citizens are taxpayers according to income distribution, but there is corrupt behaviour from public officials, then group of poor citizens is the most affected. Section 3 consider that citizens can evade paying taxes, and so being corrupt citizens, then tax authority may consider an auditing mechanism or tax control which depends on both the taxation rate and the penalty rate. Section 4 develops the network structure model for tax collection, while section 5 presents the numerical simulation on such a networks structures and its modifications. Section 6 concludes the paper.

2 Corrupt officials and taxpayers

Consider an economy where corruption is present in all public officials, and a model of tax control based on Bourne and Kumacheva (2010). There is a homogeneous population of n taxpayers, each of them has an income y_i . Without loss of generality, consider the two extreme income groups, i.e. the rich R and the poor P . The public officials always perform acts of corruption on these two populations, as they want a bribe or part of the income of these citizens. Assume that public officials may overvalue, $\bar{y} > y$ or undervalue $\underline{y} < y$, the income of the citizens, where $y > 0$ denotes the true valuation. However, this overvaluation or undervaluation depends on the true valuation. Let \bar{y} and \underline{y} be given by:

$$\bar{y}(y) = \lambda y, \quad \forall \lambda > 1. \quad (1)$$

$$\underline{y}(y) = \beta y, \quad \forall 0 < \beta < 1. \quad (2)$$

We assume that higher values of y reflect more taxation on citizens. In case the public official, as an assessor, reports \bar{y} , the taxpayer has the right to approach a court of law and appeal against the assessment. But there is a cost of doing so. The costs of proving that the right valuation is y instead of \bar{y} is given by:

$$C(y) = \alpha_0 + \alpha_1 y \quad (3)$$

$C(y)$ contains a fixed part α_0 which suggests that no matter what the value of y is, for example one has to run-around and make certain number of trips to the appellate authorities. Instead $\alpha_1 y$ says that depending on y , certain fees need to be paid to the legal expert fighting for the plaintiff. Even if some costs are reimbursed, there is always a net cost.

Definition 1. Consider a proportional tax rate, $\tau > 0$, on the true individual income, y , which cannot be misreported to the authorities (for instance formal wages, public and private salaries, shareholders, bank accounts, ecc.). An ‘honest’ system is when $y(1 - \tau)$ is the individual net pay-off.

Note that in case \bar{y} is reported, a citizen would go to the court iff the following holds,

$$y - \tau y - \alpha_0 - \alpha_1 y > y - \tau \lambda y$$

or,

$$y > \frac{\alpha_0}{\tau(\lambda - 1) - \alpha_1}$$

Which says that the benefit of getting a court-verdict must outweigh the cost of doing so. Let

$$\tilde{y} = \frac{\alpha_0}{\tau(\lambda - 1) - \alpha_1} > 0. \quad (4)$$

So, $\forall y > \bar{y}$, the taxpayer will go to the court and its reservation pay-off would be $[y(1 - \tau) - (\alpha_0 + \alpha_1 y)]$. Similarly $\forall y \leq \bar{y}$, the taxpayer citizen will not go to the court and its reservation pay-off would be $(y - \tau \lambda y)$. Notice that if someone is indifferent between the two, he chooses not to go to the court. As we mentioned there is perfect information about individual income by part of official agents. Hence public officials know these reservation pay-offs for these two groups of taxpayers, the richest one, R , and poorest one, P . Let us define these pay-offs as,

$$R \equiv [y(1 - \tau) - (\alpha_0 + \alpha_1 y)], \quad \forall y > \bar{y}$$

and

$$P \equiv (y - \tau \lambda y), \quad \forall y \leq \bar{y}$$

The public official is corrupt and behaves in the following way. He/She would like a bribe for announcing y . But if the taxpayer insists on y instead, then the corrupt official intimidate him, and so \bar{y} will be assessed. Basically the public official wants a share of $S_R > 0$ from citizen R and of $S_P > 0$ from P as a bribe, where S_R and S_P are defined by:

$$S_R = y - \tau \beta y - R \quad (5)$$

and,

$$S_P = y - \tau \beta y - P \quad (6)$$

Let us assume that some bargaining power yields σS_R and σS_P to the public official, $0 < \sigma < 1$, i.e. it is the bargaining power or how much power corrupt officials have to take over for appropriating some part of the wealth of the people.

Therefore, the net pay-off to R and P citizens are,

$$\Pi_R = y - \tau \beta y - \sigma S_R \quad (7)$$

and,

$$\Pi_P = y - \tau \beta y - \sigma S_P \quad (8)$$

We are now in a position to compare Π_R and Π_P with $y(1 - \tau)$, the net pay-off in a honest system (Definition 1). That is,

$$\Pi_R - y(1 - \tau) = \tau y(1 - \beta) - \sigma[\tau y(1 - \beta) + (\alpha_0 + \alpha_1 y)],$$

hence:

$$\Pi_R > y(1 - \tau) \iff \sigma < \frac{\tau y(1 - \beta)}{\tau y(1 - \beta) + (\alpha_0 + \alpha_1 y)}, \quad (9)$$

and:

$$\Pi_P - y(1 - \tau) = \tau y(1 - \beta) - \sigma \tau y(\lambda - \beta)$$

hence:

$$\Pi_P > y(1 - \tau) \iff \sigma < \frac{1 - \beta}{\lambda - \beta}. \quad (10)$$

What (9) and (10) show, is to indicate how people with varying levels of income (or imputed income), threatened by the corruption of officials under a bribery behavior, compare their position vis-a-vis the honest system versus the “corrupt-system”. Let us state the following proposition.

Proposition 1. *Rich and poor citizens prefer an honest system or a corrupt system based on:*

- For $\sigma \leq \left(\frac{1-\beta}{\lambda-\beta}\right)$, everyone prefers the corrupt system to the honest system.
- For $\sigma \in \left(\frac{(1-\beta)}{(\lambda-\beta)}, \frac{\tau(1-\beta)}{\tau(1-\beta)+\alpha_1}\right) \exists y^* > \tilde{y}$ such that people with $y < y^*$ prefer the honest system to the corrupt state and people with $y > y^*$, prefer the corrupt system to the honest system.
- For $\sigma \in \left[\frac{\tau(1-\beta)}{\tau(1-\beta)+\alpha_1}, 1\right]$, everyone prefers the honest system.

Proof. Let us prove the above statement for each item.

- If $\sigma \leq \left(\frac{1-\beta}{\lambda-\beta}\right)$, from condition (10) it is obvious that $\forall y \leq \tilde{y}$, corrupt system will be preferred, since $\forall y > \tilde{y}$, we get that:

$$\frac{\tau(1-\beta)}{\tau(1-\beta) + \alpha_0/y + \alpha_1} > \frac{(1-\beta)}{(\lambda-\beta)},$$

and then everyone will prefer the corrupt system.

- Note that as $y \rightarrow \infty$, the RHS of (9), i.e. $\frac{\tau(1-\beta)}{\tau(1-\beta) + \alpha_0/y + \alpha_1}$ tends to $\frac{\tau(1-\beta)}{\tau(1-\beta) + \alpha_1}$. If $\frac{(1-\beta)}{(\lambda-\beta)} < \sigma < \frac{\tau(1-\beta)}{\tau(1-\beta) + \alpha_1}$ exists y^* such that $\frac{\tau(1-\beta)}{\tau(1-\beta) + \alpha_0/y + \alpha_1} = \sigma$, where $y^* > \tilde{y}$. Hence **for all** $y \leq y^*$ people will prefer the honest system, whereas for $y > y^*$, people will prefer the corrupt system.
- Here, along with (9), (10) is reversed for all possible y . Therefore, everyone prefers the honest system.

Thus it has been demonstrated. \square

Corollary 1. *Thus the richer section may prefer a corrupt system compared to the one where the public official behaves honestly.*

Notice that, in the general case with given $\alpha_0 > 0$, $\alpha_1 > 0$, people with very high y , if corrupted by public officials, will go to the court and gets $y(1 - \tau) - (\alpha_0 + \alpha_1 y)$. But he/she can share a surplus $\tau y(1 - \beta) + (\alpha_0 + \alpha_1 y)$ with corrupt officials. So, unless σ is high enough, his/her net pay-off is greater than $y(1 - \tau)$ which he/she gets in the ‘honest’ system. So he/she prefers the corrupt system. However, this depends on the magnitude of α_1 and σ . If $\alpha_1 = 0$, there is no $\sigma < 1$ for which everyone prefers the honest system. Moreover, for the same $\sigma > \frac{(1-\beta)}{(\lambda-\beta)}$, critical y^* will go down.

For richer people the costs for corrupt behavior are relatively low and the average cost for facing corruption goes down with the level of income as there is a fixed cost. Hence, the richer section has a stronger bargaining power while sharing the benefit of underreported income relative to those who are poor and face relatively high costs for facing corruption.

3 Corrupt behaviour of taxpayers

Let us analyze the other side of the coin, that is when taxpayers cheat in terms of their true income, and thus try to evade taxes. As before, every citizen has true income y_i , but now he/she cheats and declares an income $y^* \leq y$ after each tax period. Once again, the total set of taxpayers is divided into the groups of low level income agents, the poor, and high level income agents, the rich.¹

For every taxpayer i , their incomes can take only two values: $y_i \in \{P, R\}$, where P is the poor citizen or low-level income agent and R is the rich citizen or high-level income agent ($0 \leq P < R$). Declared (false, cheated) income y_i^* also can take values from the mentioned binary set $y_i^* \in \{P, R\}$. Thus, in this model there are three different strategies $y_i^*(y)$, which depend on the relation between true and declared incomes:

1. $y_i^*(y) = P(P)$;
2. $y_i^*(y) = R(R)$;
3. $y_i^*(y) = P(R)$.

Obviously that the taxpayers from the first and the second groups declare their income correspondingly to its true level and they do not intend to evade. The third group is the group of tax evaders.

Those rich citizens who declare themselves as poor, hence this group is of interest of the tax authority. The tax authority audits those taxpayers, who declared $y_i^*(y) = P$, with the probability $T_P \in [0, 1]$ in every tax period.

Let's suppose that tax audit is absolutely effective, i. e. it reveals the existing evasion. The proportional case of penalty is considered, and there is a penalty rate $\mu > 0$. Once the tax evasion is revealed, the evader must pay $(\tau + \mu)(y_i - y_i^*) > 0$, where constants τ and μ are the tax and the penalty rates correspondingly, and $(y_i - y_i^*)$ is the hidden income. For the agents from the studied groups the payoffs are given by:

$$\pi(P(P)) = (1 - \tau)P; \quad (11)$$

$$\pi(R(R)) = (1 - \tau)R; \quad (12)$$

$$\pi(P(R)) = R - \tau P - T_P(\tau + \mu)(R - P). \quad (13)$$

The tax authority gets information about taxpayers' incomes from their tax declarations and audits those, who declared $y_i^* = P$. The fraction of audited taxpayers is T_P . It's obvious that either the agents from the first group (who actually have true income $y_i = P$) or the evaders from the third group are both in this fraction of audited taxpayers.

The total set of the taxpayers is divided into the following groups: wealthy taxpayers, who pay taxes honestly ($y^*(y) = R(R)$), insolvent taxpayers ($y^*(y) = P(P)$) and wealthy evaders ($y^*(y) = P(R)$).

Hence, the following arguments, related to the searching of possible tax evasions, apply to the third group of the agents, declared $y^*(y) = P(R)$. The following proposition is straightforward.

Proposition 2. *If taxpayers are risk neutral, then the optimal value of audit probability depends on taxes, τ , and inversely on the penalty rate, μ .*

Proof. Risk neutral taxpayers' behaviour supposed to be rational, i.e. their tax evasion is impossible only if the risk of punishment is so high that the tax evader's profit is less or equal to his/her expected post-audit payments (in the case when his evasion is revealed), then:

$$T_P(\tau + \mu)(R - P) \geq \tau(R - P).$$

¹Note that the number of partitions can be increased, but it does not effect on the following arguments and conclusions.

Therefore, the critical value of audit probability T_P (due to the taxpayer's decision to evade or not) is

$$T_P^* = \frac{\tau}{\tau + \mu}. \quad (14)$$

For this type of models the optimal solution is usually presented in the form of the “threshold rule” in various modifications (see, for example, Chander and Wilde (1998) or Vasin and Morozov (2005)). In Boure and Kumacheva (2010) this rule is formulated so that the optimal value T_P^* of the auditing probability is defined from (14), and for the risk neutral taxpayer the optimal strategy is

$$y^*(y) = \begin{cases} R, & T_P \geq T_P^*; \\ P, & T_P < T_P^*. \end{cases} \quad (15)$$

In our study we assume that people from risk-loving and risk-averse subgroups keep their behavior despite of received information. It means that the value of collected taxes does not depend on these groups, whereas risk-neutral agents react on the received information and can change their behavior. For example if a risk-neutral agent receives information that probability of tax audit is high then he/she pays taxes honestly, else he/she evades. \square

We assume that information can be disseminated through social networks. We may consider it as a “stochastic alarm” (see Sandholm, 2010), which means the signal to agent that he/she has an opportunity to react to the changes in environment. Nevertheless there are some problems which should be fixed to make the static model described above close to real-life process. The first problem is that the players are supposed to be risk neutral. However in real life there are also risk averse and risk loving economic agents. Another problem is that we consider the game with complete information. It is assumed that the taxpayers know (or can estimate) the value of the auditing probability, but in current study we do not take into account the method of receiving information. Another problem is that the auditing with optimal probability (14) is excessively expensive and the tax authority usually has strongly limited budget, thus the actual value of T_P should be substantially less than T_P^* in real life. By taking into consideration all mentioned reasons, in what follows, we formulate an extended model of tax auditing which includes an information component and an evolutionary process of adaptation of population of taxpayers to changes in the economic environment.

4 Evolutionary games, networks and tax collection

In this section we introduce the formulation of evolutionary model of information spreading over population of taxpayers. In a real life situation, agents who received information will share it with their closest neighbors, such as family, friends, and colleagues. Hence it is more natural to consider a population with a network structure in formalization of evolutionary game. The main difference from the classical evolutionary game is that here agents transfer information not to a random opponent but they communicate only with connected neighbors and friends. In this case we can describe the possible links between agents by using network analysis.

Let $G = (N, L)$ denote an indirect network, where $N = \{1, \dots, n\}$ is a set of economic agent and $L \subset N \times N$ is an edge set. Each edge in L represents two-player symmetric game between connected taxpayers. The taxpayers choose strategies from a binary set $X = \{\text{to pay taxes, to evade taxes}\}$ and receive payoffs according to a matrix of payoff. Each instant time moment agents use a single strategy against all opponents and thus the games occurs simultaneously. We denote the strategy state by $x(t) = (x_1(t), \dots, x_n(t))^T$, $x_i(t) \in X$. Here $x_i(t) \in X$ is a strategy of taxpayer i , $i = \overline{1, n}$, at time moment t . Aggregated payoff of agent i can be defined as in Riehl and Cao (2015), i.e.

$$u_i = \omega_i \sum_{j \in M_i} a_{x_i(t), x_j(t)}, \quad (16)$$

where $a_{x_i(t), x_j(t)}$ is a component of payoff matrix, $M_i := \{j \in L : \{i, j\} \in L\}$ is a set of neighbors for taxpayer i , weighted coefficient $\omega_i = 1$ for cumulative payoffs and $\omega_i = \frac{1}{|M_i|}$ for averaged payoffs. Vector of payoffs of the total population is $u(t) = (u_1(t), \dots, u_n(t))^T$.

The state of population will be changed according to the rule, which is a function of the strategies and payoffs of neighboring agents:

$$x_i(t+1) = f(\{x_j(t), u_j(t) : j \in N_i \cup \{i\}\}). \quad (17)$$

Here we suppose that taxpayer changes his/her behavior if at least one neighbor has better payoff. As the example of such dynamics we can use the proportional imitation rule (Sandholm, 2010), in which each agent chooses a neighbor randomly and if this neighbor received a higher payoff by using a different strategy, then the agent will switch with a probability proportional to the payoff difference. The proportional imitation rule can be presented as:

$$p(x_i(t+1) = x_j(t)) := \left[\frac{\lambda}{|M_i|} (u_j(t) - u_i(t)) \right]_0^1 \quad (18)$$

for each agent $i \in L$ where $j \in M_i$ is a uniformly randomly chosen neighbor, $\lambda > 0$ is an arbitrary rate constant, and the notation $[z]_0^1$ indicates $\max(0, \min(1, z))$.

As a game theoretical model between connected taxpayers we use one of the bimatrix games with known structure.

Remark 1. *Corruption and evasion of taxation can be analyzed as baseline games, i.e. i) prisoner's dilemma game, ii) stag hunt game, and iii) hawk-dove game. Whereas total tax revenue depends on information regarding tax control system.*

Below we present the games in terms of tax-control system. As in classical evolutionary game theory, the instant communications between taxpayers is described by two players bimatrix game. We denote by A a matrix for the first player and the payoff matrix for her opponent by $B = A^T$. The next modified payoff matrix is adapted to the model of tax control.

The Prisoner's Dilemma As well it was shown in [49], symmetric bimatrix games can be classified in four classes, where each class characterized by the structure of the game and general properties of strategies and equilibrium profiles. That is, for all games we define the payoff matrix A for the first player and since we have symmetric games then the payoff matrix B for the second player is $B = A^T$. For the Prisoner's Dilemma the matrix A will take a form

C	$\bar{u} + SW$	$\bar{u} - SW$
D	$u(L(H))$	\bar{u}

where strategy C means to cooperate (to pay taxes) and strategy D – to defect (to evade), payoff $\bar{u} = 1/2u(P(P)) + 1/2u(R(R))$ is an average profit of the “mean” agent, which means that we can play with the equal probability with an opponent with low level of income and with high level of income. Let a parameter $SW > 0$ be a social welfare level, obtained for the participation in social consolidation, which means that if taxpayers pay taxes then he/she receives some social goods provided by the collected taxes.

The Stag Hunt game. Since this bimatrix game describes a social choice between personal and public goods then we choose it as an alternative instant game between connected taxpayers. Here payoff matrix for the first player is

S	$\bar{u} + SW$	0
H	$\bar{u} - SW$	$\bar{u} - SW$

where we can interpret players strategies in the following way: strategy S and H correspond to hunt and a hare in classical case. In our framework S means to pay taxes and H means to evade respectively.

The Hawk-Dove game. At the third variant of the instant game we use the Hawk-Dove game, which also describes a conflict between more and less aggressive players. In our case we define a payoff matrix for the first player as:

$$\begin{array}{c|cc} & H & D \\ \hline H & \frac{u(P(R)) - (\tau + \mu)(R - P)}{2} & \frac{\bar{u} + SW}{2} \\ D & 0 & \frac{\bar{u} + SW}{2} \end{array}$$

where strategy H corresponds to be a Hawk, that means to use an aggressive behavior, i.e. to evade, and strategy D corresponds to be a Dove that means to use a passive behaviour, i.e. to pay taxes. Here we suppose that individual payoffs satisfy the inequality $u(P(R)) < (\tau + \mu)(R - P)$, which means that it works for the large values of parameters τ and μ or when there is a big difference $(R - P)$.

4.1 Total Tax Revenue

We use the next assumptions to describe the aggregated system costs which occur during the process of information spreading over the population of taxpayers:

- The considered population is restricted by the subpopulation of rich taxpayers R with high level of income, where n_R is a number of agents in population;
- If there is no information circulates in the populations then the total population evades hence $n_R = n_{ev}$, where n_{ev} is the number of evaders);
- If information is injected in the initial time moment by the tax authority then we denote by $n_{inf}^0 = n_{nev}^0$ the number of informed taxpayers about increased probability of tax auditing and then they decided not to evade;
- In each time moment we have $n_R = n_{ev}(t) + n_{nev}(t)$ (or $\nu_{nev}(t) + \nu_{ev}(t) = 1$), $t \in [0, T]$, where T is the time of information injection.

Based on these assumptions we can state the following corollary.

Corollary 2. *There are two different types of the model for tax control. The first model does not include the process of information dissemination. If information does not circulate in population then risk-neutral taxpayers do not pay taxes. Also if taxpayers suppose that the probability of auditing is rather small ($T_p < T_p^*$), then they also evade and the only honest taxpayers are risk averse agents.*

Hence, we are able to compute the total tax revenue in case of absence of information, i.e.

$$TTR_0 = n_P \tau P + n_R (\tau P + T_P (\tau + \mu)(R - P)) - n T_P c. \quad (19)$$

If we take into account the dissemination of information in the population of taxpayers then at the initial moment tax control system injects an information into the population. Since information starts to circulate in population then we have a share of informed taxpayers $\nu_{inf}^0 = \nu_{inf}(t_0)$. The cost of unit of information is c_{inf} ($c_{inf} < c$). At the moment when the system reached its steady state ν_{inf} is the share of those who perceived information and paid taxes, ν_{ev} is the share of those who still evades. In this case the total tax revenue is

$$TTR_T = n_P \tau P + n_R (\nu_{nev}^T \tau R + \nu_{ev}^T (\tau P + T_P (\tau + \mu)(R - P))) - n(T_P c + \nu_{inf}^0 c_{inf}), \quad (20)$$

where ν_{nev}^T is the share of taxpayers who don't evade at the moment $t = T$, ν_{ev}^T is the share of taxpayers who continue to evade taxation at the moment $t = T$, ν_{inf}^0 is the value (fraction) of the informational injection at the initial time moment ($\nu_{inf}^0 = \nu_{inf}(t_0)$).

In the following section, we present a simulation analysis on dynamic networks to verify the aforementioned about the control of tax collection, considering: i) tax and penalty rates, ii) probabilities and costs of auditing, iii) the information levels, and so iv) looking at the network's dynamics of the share of tax evaders citizens.

5 Numerical simulations

In this section we present series of numerical experiments to depict the results and illustrate the changes in the population of taxpayers over the time. In all experiments we have used the following data for tax and penalty rates (tabl. 1) and the distribution of incomes over the total population, which have been taken from the [Russian Federation State Statistics Service, 2018].

Table 1: General parameters for numerical simulations.

Parameter	Value
tax rate	$\tau = 13\%$
penalty rate	$\mu = 13\%$
optimal value of the probability of audit	$T_P^* = 0.5$
actual value of the probabilities of audit	$T_P = 0.1$
unit cost of auditing	$c = 7455$ (rub.)
unit cost of information injection	$c_{inf} = 10\%c = 745.5$ (rub.)
stopping point of the iteration process	$\sqrt{\sum_{i=1}^n (x_i(t) - x_i(t+1))} \leq 10^{-3}$

The distribution of the income among the population of Russian Federation in 2017 (see Tab. 2).

Table 2: Two modeled groups and average income

group	income (rub./month)	interval	average (rub.)	income	share of population (%)
P	less 25000		$P = 12500$		43
R	more 25000		$R = 50000$		57

To visualize the results of iterative process of spreading information over the population of taxpayers we use a small network of 25 nodes ($n_R = 25$), however all these results are also correct for a large number of nodes. By using formula of total tax revenue (19) we receive $TTR_0 = 56855.81$ rub in each experiment. In all figures we mark the agents who use strategy "to pay taxes" as yellow dots and the agents who use

strategy “to evade” as blue dots. In simulations we use different modification of graphs such as strongly connected network, weakly connected network, random graph and grid. The division of the experiment into different series is also determined by the network configuration. The results of network simulations are presented in the following tables. We use the next notations: PD is the Prisoner’s Dilemma, HD is the Hawk-Dove game, SH is the Stag Hunt game.

5.1 Series 1: Simulation for Grid

The results of experiments for the grid configuration are summarized and presented in the table 3. Their graphic illustrations can be seen in the Figs. 1 – 3. **In all experiments we receive the shares of evaders u_{ev} and non-evaders n_{nev} taxpayers in population in the initial and final time moments.**

Table 3: Results of Simulation for Grid

Num- ber	Initial injection of information ν_{inf}^0	Initial state (n_{nev}^0, n_{ev}^0)	Payoff matrix	Number of iterations	Final state (n_{nev}^T, n_{ev}^T)	The value of TTR_T (rub)
1	15	(15, 10)	PD	6	(0, 25)	24 071.12
2	11	(11, 14)	SH	2	(11, 14)	49 306.94
3	4	(4, 21)	HD	25	(25, 0)	87 592.56

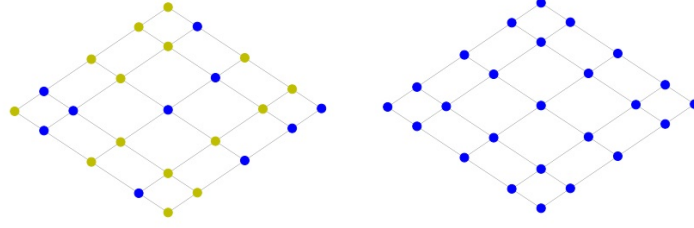


Figure 1: Initial state: $(n_{nev}, n_{ev}) = (15, 10)$; final state: $(n_{nev}, n_{ev}) = (0, 25)$.

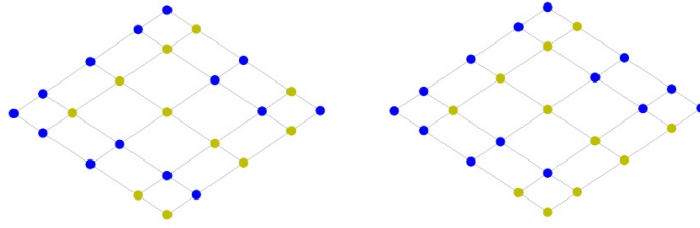


Figure 2: Initial state: $(n_{nev}, n_{ev}) = (11, 14)$; final state: $(n_{nev}, n_{ev}) = (11, 14)$.

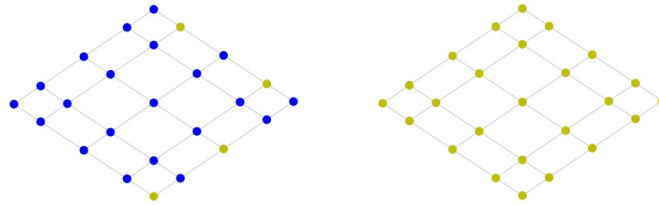


Figure 3: Initial state: $(n_{nev}, n_{ev}) = (4, 21)$; final state: $(n_{nev}, n_{ev}) = (25, 0)$.

5.2 Series 2: Simulation for Random Graph

The results of experiments for the random graph configuration are presented in the Tab. 4, the graphic illustrations are in the figure 4.

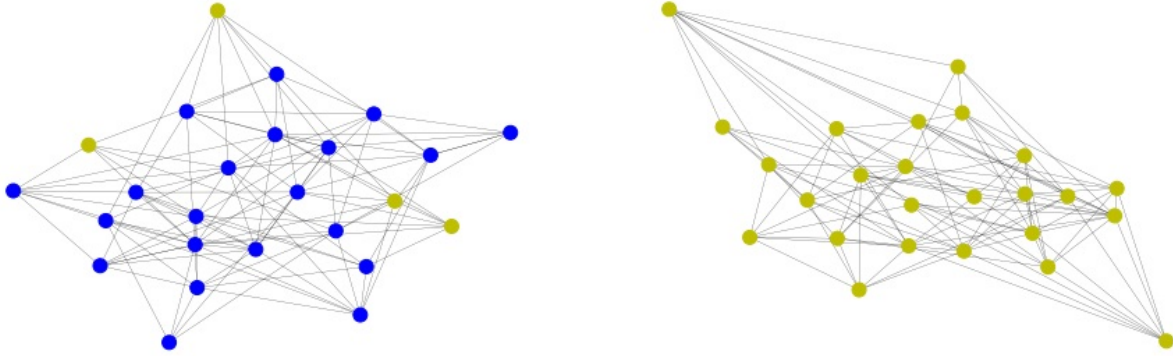


Figure 4: Initial state: $(n_{nev}, n_{ev}) = (4, 21)$; final state: $(n_{nev}, n_{ev}) = (25, 0)$.

Table 4: Results of Simulation for Random Graph

Num- ber	Initial injection of information ν_{inf}^0	Initial state (n_{nev}^0, n_{ev}^0)	Payoff matrix	Number of iterations	Final state (n_{nev}^T, n_{ev}^T)	The value of TTR_T (rub)
1	4	(4, 21)	HD	18	(25, 0)	87 592.56

5.3 Series 3: Simulation for Strongly Connected Network

The results of experiments for the strongly connected network are summarized and presented in the Tab. 5. Their graphic illustrations are presented in Figs. 5 – 7.

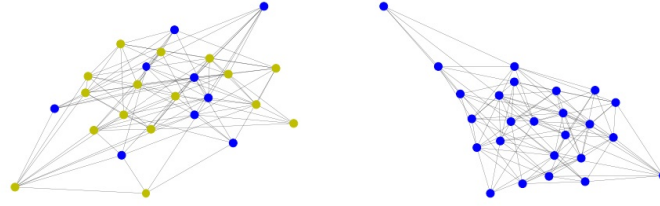


Figure 5: Initial state: $(n_{nev}, n_{ev}) = (16, 9)$; final state: $(n_{nev}, n_{ev}) = (0, 25)$.

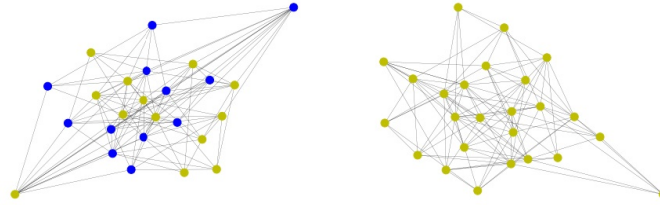


Figure 6: Initial state: $(n_{nev}, n_{ev}) = (13, 12)$; final state: $(n_{nev}, n_{ev}) = (25, 0)$.

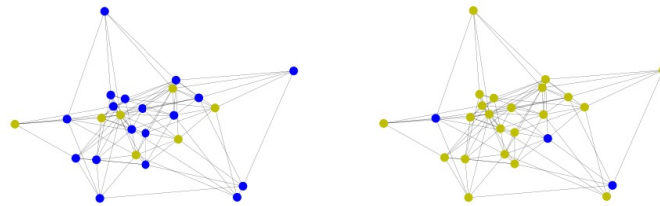


Figure 7: Initial state: $(n_{nev}, n_{ev}) = (7, 18)$; final state: $(n_{nev}, n_{ev}) = (22, 3)$.

Table 5: Results of Simulation for Strongly Connected Network

Number	Initial injection of information ν_{inf}^0	Initial state (n_{nev}^0, n_{ev}^0)	Payoff matrix	Number of iterations	Final state (n_{nev}^T, n_{ev}^T)	The value of TTR_T (rub)
1	16	(16, 9)	PD	8	(0, 25)	19 598.12
2	13	(13, 12)	SH	7	(25, 0)	80 883.06
3	7	(7, 18)	HD	11	(22, 3)	78 270.25

5.4 Series 4: Simulation for Weakly Connected Network

The results of experiments for weakly connected network are summarized and presented in the Tab. 6. Their graphic illustrations can be seen in Figs. 8 – 9.

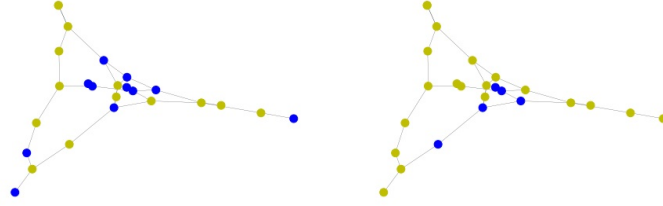


Figure 8: Initial state: $(n_{nev}, n_{ev}) = (14, 11)$; final state: $(n_{nev}, n_{ev}) = (20, 5)$.

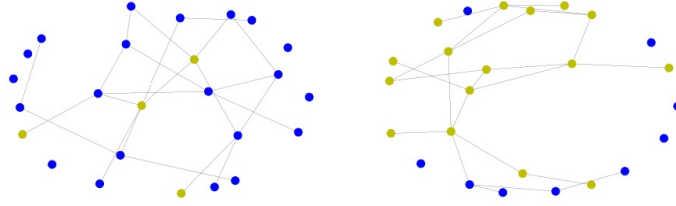


Figure 9: Initial state: $(n_{nev}, n_{ev}) = (4, 21)$; final state: $(n_{nev}, n_{ev}) = (16, 9)$.

Table 6: Results of Simulation for Weakly Connected Network

Num- ber	Initial injection of information ν_{inf}^0	Initial state (n_{nev}^0, n_{ev}^0)	Payoff matrix	Number of iterations	Final state (n_{nev}^T, n_{ev}^T)	The value of TTR_T (rub)
1	14	(14, 11)	SH	4	(20, 5)	68 327.87
2	4	(4, 21)	HD	6	(16, 9)	66 335.12

5.5 Numerical simulations: the main trends and discussion

The series of numerical experiments show that the final distribution of taxpayers between honest and evaders depends on the some reasons. We receive that network structure and an instant game between connected taxpayers strongly influence on the final state. In our simulation we have that if we choose the Prisoner's Dilemma as an instant game then due to the properties of this game, evaders, which strategy corresponds to strategy D prevail. That means that the equilibrium is achieved in the case when both interacting agents choose a strategy of tax evasion. If we use the Stag Hunt game as a game between connected taxpayers then we have several subcases:

- If we calculate cumulative and average payoffs with uniform initial distribution of strategies then in the steady state the honest taxpayers prevail.
- However we receive that both mixed equilibria are possible, as well as situations where all taxpayers at the final moment of time shy away or not evade taxes.

For the Hawk-Dove game as an instant game we have that mixed equilibrium or equilibrium of non-evaders prevail. There is a positive trend from the fiscal authorities' point of view: the increasing of the number of non-evading agents.

Additionally we can say that the final distribution of taxpayers in iterative process of information spreading depends on: the number of iterations, existence of isolated groups of taxpayers who are not involved in propagation process; the initial injection of the information (the share of those who received the information); class of payoff matrix which influence on the structure of Nash equilibria.

As the main results we obtain that the propagation information about possible tax audit over papulation of taxpayers brings a positive effect for the total revenue of fiscal system and increases total amount of honest taxpayers. Knowledge of the structure of payoff matrix simplifies the behavioral analysis of the impact of information on the effectiveness of tax control.

6 Concluding remarks

Corruption on the part of the bureaucracy appears or operates in several ways, but above all in two directions: diversion of resources in non-existent firms; or in work contracts drafted and designed with skill to economically favor one or both parties, i.e. agreement between government and private initiative at all levels. From this perspective, public expenditure, as an integral part of national income, is diverted and therefore only a small part of the population, in general the richest, is benefited, but above all because as a positive multiplier effect of the economy, its application distorted that impact, reducing aggregate income and therefore aggregate demand, the domestic market. In the absence of efficiency in public administration, plus impunity between the parties involved in corruption, for the rich it is easier to resort to bribery or collusion, both to reduce processing times, and to conduct business with guaranteed profit. Although corruption is an important factor in the inequitable distribution of income, as the model demonstrates, and therefore a factor that can influence the reduced economic growth, although the streamlining of procedures through bribery could eventually boost business. However, inequality is one of the most visible effects of corruption.

In this research paper we study the network's dynamics of agents with High and Low level of income (Poor and Rich agents), where these two groups of agents differ by their relation to corruption on a tax payment system. It is known that audit of whole population of taxpayers needs large budget to catch taxpayers who prefer to evade taxes and reach an optimal value of audited subpopulation. However, usually tax authority has limited budget and it is necessary to find additional methods to reach optimal share of audited taxpayers.

Therefore, we have included the process of information spreading into an evolutionary model of tax control on the network. Evolutionary model helps to estimate the impact of information about future tax audit, received from the tax authority on the decisions of taxpayers. We also reformulate classical bimatrix games in terms of tax authority system to use them as an instant games into evolutionary process. Series of numerical simulations have shown that the final distribution of taxpayers who pay taxes depends on the network structure and received information. Hence propagation information about possible tax audit gives a positive effect for the total revenue of fiscal system and increases total amount of taxpayers who prefer to pay taxes honestly. The process of changes of taxpayers' behavior over the time is presented by the complicated model we illustrate in numerical experiments.

Tax authorities disseminate information that their intention is to verify no less than this proportion of the population. However, due to the fact that the budget is not always such as to provide such an audit plan, this information in particular may turn out to be misinformation. But even in this case, it is economically efficient (increases tax collection), which we showed **using** our simulation modeling. All experiments were made under the assumption of full rationality of agents, which means that we take into account only risk-neutral agents, but in future it is possible to extend the model with risk-loving and risk-averse agents.

But, as has been shown, one of the most important aspects for restarting the momentum of economic growth and the distribution of wealth for development, among other strategies of effective economic policy, is to reduce corruption and, above all, to have effective control of the ability to collect taxes to apply a policy for the redistribution of income.

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