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Optimal Incentives in a Principal-Agent Model with Endogenous Technology*

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Abstract

One of the standard predictions of the agency theory is that more incentives can be given to agents with lower risk aversion. In this paper we show that this relationship may be absent or reversed when the technology is endogenous and projects with a higher efficiency are also riskier. Using a modified version of the Holmstrom and Milgrom's (1987) framework, we obtain that lower agent's risk aversion unambiguously leads to higher incentives when the technology function linking efficiency and riskiness is elastic, while the risk aversion-incentive relationship can be positive when this function is rigid.

Keywords: principal-agent; incentives; risk aversion; endogenous technology.
JEL Classification: D82.

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1 Introduction

One of the main features of the agency theory is the trade-off between incentives and insurance. A standard result is that, other things equal, more uncertainty should increase the gains from insuring the agent and reduce the pay-for-performance sensitivity. This is because pay-for-performance contracts induce the (risk-averse) agent to exert more effort but also imply higher wage costs when risk increases. Moreover, in the standard principal-agent framework, a lower risk aversion of the agent allows the principal to provide more incentives by making the payment of the agent more related to output.

The empirical works testing the link between uncertainty and the provision of incentives have found mixing results (e.g., Rao and Hanumantha, 1971; Allen and Lueck, 1995; Aggarwal and Samwick, 1999; Core and Guay 2002; Wulf, 2007). In many cases, the empirical findings are even in contradiction with the standard predictions of the theory as they document a positive (rather than negative) correlation between observed measures of uncertainty and the provision of incentives (see Prendergast, 2002, for an extensive discussion on this point).

In this paper we analyze the effect of variations in the agent's risk aversion on the pay-for-performance sensitivity when the principal can choose among many technologies (or projects) that differ for their riskiness and efficiency. In particular, we use a modified version of the principal-agent framework of Holmstrom and Milgrom (1987) where the principal also chooses the technology employed and riskier technologies are assumed to be more efficient.

We obtain that a lower risk aversion of the agent has not only the standard direct positive effect on incentives but it also generates an indirect effect on the pay-for-performance sensitivity through the change of the technology employed. Indeed, a lower agent's risk aversion makes it optimal for the principal the adoption of a riskier and a more efficient technology. While the higher efficiency of the technology allows the principal to give more incentives to the agent (so reinforcing the direct effect), its higher riskiness makes the provision of incentives more costly which works in the direction of reducing the optimal degree of the pay-for-performance sensitivity. Therefore, the final effect of the agent's risk aversion on incentives will generally be ambiguous.

We then characterize the conditions under which the sign of this relationship is definite. When the technology function describing the link between riskiness and efficiency is elastic, the effect on incentives generated by the higher efficiency of the new technology is greater than the one induced by the higher riskiness. Therefore, the net indirect effect reinforces the direct effect and leads to an increase of the share of output paid to the agent. When instead the technology function is rigid, the effect on incentives due to higher riskiness of the new technology dominates the one caused by the increased efficiency as the increased efficiency is small relative to the increased riskiness. In this case, the final effect of risk aversion on the pay-for-performance sensitivity will generally be ambiguous; and when the increased riskiness of the new technology is sufficiently strong, a lower risk aversion may lead to a decrease in pay-for-performance sensitivity.

This paper is closely related to the strand of the literature on endogenous matching between a principal and an agent. Some papers (e.g., Legros and Newman, 2007; Serfes, 2005, 2008; Wright, 2004) obtain that more averse agents should end up matching with riskier firms in equilibrium. Li and Ueda (2009) propose an endogenous matching model where firms have different levels of riskiness and agents differ in productivity. They obtain that safer firms offer high-powered incentives schemes and riskier firms' contracts are characterized by a lower pay-for-performance sensitivity. The result is that in equilibrium safer firms should be matched with more productive agents.

Our paper is also related to the literature investigating the link between risk aversion and incentives. For example, Grund and Sliwka (2010) find evidence that a higher agent's risk aversion has a negative impact on the probability that the payment scheme is performance contingent. Recent laboratory experiments (e.g., Cadsby et al. 2009) highlight the fact that usually agents adapt efforts to reduce their risk exposure affecting, in such a way, their final productivity. This confirms that the principal may have an interest to select different technologies to cope with different degrees of agent's risk aversion.

The paper is organized as follows. In Section 2 we describe the framework and Section 3 provides the solution of the model. Section 4 presents the comparative statics analysis of the effect of a reduction of the agent's risk aversion on incentives. Section 5 concludes.

2 The Framework

We consider a moral hazard model as in Holmstrom and Milgrom (1987). The principal owns the technology and is risk neutral. The agent is risk averse and has a constant absolute risk aversion (CARA) utility function with a coefficient of absolute risk aversion equal to r. Total output is equal to

$$y = e + \varepsilon, \tag{1}$$

where e is the agent's action (e.g., effort) and ε is an (unobservable) random variable normally distributed with zero mean and variance σ^2. The technology is characterized by quadratic costs, so that the agent's cost of action is

$$c(e) = \frac{k}{2}e^2, \tag{2}$$

where k is a constant representing the efficiency of the technology employed. Better technologies are characterized by a lower k and vice-versa. The agent's reservation utility is equal to δ.

We here modify the Holmstrom and Milgrom's framework by assuming the existence of a given set of technologies (or projects) with different levels of efficiency and riskiness among which the principal can choose. In particular, we assume a trade-off between efficiency and riskiness so that technologies with a higher volatility σ^2 also have a lower marginal cost of effort, i.e.,

$$k \equiv k(\sigma^2) \;\; \text{with} \;\; k' \equiv \frac{dk}{d\sigma^2} < 0, \tag{3}$$

where $k > 0$ for all $\sigma^2 \in (0, \infty)$. For simplicity, $k(\cdot)$ is assumed to be a function continuous and differentiable in σ^2.

In this framework, the principal decides the optimal technology and the agent's payment scheme; then, the agent optimally chooses the action. In the next sections, we determine these choices and analyze the effects of a variation of the agents' risk aversion on the optimal payment scheme of the agent.

3 The Equilibrium

We solve the problem by determining the optimal payment scheme and the agent's action for a given technology.[1] Then, we determine the optimal technology choice of the principal.

Holmstrom and Milgrom (1987) show that a linear payment is optimal in the above framework, so that the agent's payoff can be written as $s(y) = \beta y + \alpha$, where α and β are constants optimally chosen by the principal that have to be determined. Taking into account (1), (2) and the distribution of the shock, the agent's expected utility is

$$E\left\{-\exp\left\{-r\left[s\left(y\right)-c\left(e\right)\right]\right\}\right\} = -\exp\left\{-r[\beta e + \alpha - (1/2)ke^2 - (1/2)r\beta^2\sigma^2]\right\},$$

and therefore his maximization problem can be written as

$$\max_{e} \;\; \beta e + \alpha - (1/2)ke^2 - (1/2)r\beta^2\sigma^2. \tag{4}$$

The first order condition of this problem is $\beta = ek$. Substituting this condition into (4) and then setting the expression (the agent's certainty equivalent) equal to δ gives $\alpha = -(1/2)ke^2 + (1/2)r\beta^2\sigma^2 + \delta$. Hence, the principal's maximization problem becomes

$$\max_{e} \;\; \pi = E[y - s(y)] = e - (1/2)ke^2 - (1/2)rk^2e^2\sigma^2 - \delta, \tag{5}$$

which gives the following well-known second best solution for the agent's action[2]

$$e^* = \frac{1}{k(1 + rk\sigma^2)}. \tag{6}$$

[1] We here omit some details of the analysis as the complete description of the solution can be found in Holmstrom and Milgrom (1987).

[2] The first order condition of problem (5) is $d\pi/de = 1 - ke - rk^2e\sigma^2 = 0$ and the second order condition is always satisfied as $d^2\pi/de^2 = -k - rk^2\sigma^2 < 0$.

Using the fact that $\beta = ek$, it follows that the optimal share of output paid to the agent is

$$\beta^* = \frac{1}{1 + rk\sigma^2}, \tag{7}$$

and the optimal fix payment is

$$\alpha^* = \frac{-1 + rk\sigma^2}{2k(1 + rk\sigma^2)^2} + \delta. \tag{8}$$

Let now σ_*^2 denote the variance of the optimal project. This is the solution of the following maximization problem of the principal

$$\max_{\sigma^2} \ \pi^* = \frac{1}{2k(1 + rk\sigma^2)} - \delta, \tag{9}$$

subject to the technological constraint (3), and where the maximized expect profit π^* (for a given technology) is obtained from the substitution of (6) into (5).

The first order condition of this problem is

$$\frac{d\pi^*}{d\sigma^2} = -\frac{k' + 2rkk'\sigma^2 + rk^2}{2k^2(1 + rk\sigma^2)^2} = 0, \tag{10}$$

and therefore the variance σ_*^2 of the optimal project is implicitly defined by the following equation

$$F \equiv -k' - 2rkk'\sigma^2 - rk^2 = 0, \tag{11}$$

where $k \equiv k(\sigma^2)$ and $k' \equiv k'(\sigma^2)$. The effort cost parameter at the optimal technology follows from (3) and it is $k(\sigma_*^2)$.[3]

In order to have unique interior solution, which will be useful for the comparative static analysis, we restrict the attention to functions of the technology $k(\sigma^2)$ such that F in (11) is strictly concave. This requires that the following condition is always satisfied

$$dF/d\sigma^2 = -4rkk' - 2r(k')^2\sigma^2 - k''(1 + 2rk\sigma^2) < 0. \tag{12}$$

The first component of (12) is positive (as $k' < 0$), the second is negative while the third one has the opposite sign of k''. Therefore, while $k(\sigma^2)$ can generally be concave or convex, a *sufficient* condition for (12) to hold is that k is sufficiently convex, i.e., that k'' is positive and large enough.

The following proposition summarizes these results.

Proposition 1 *The principal chooses the technology with the variance σ_*^2 implicitly defined by equation (11) and efficiency $k(\sigma_*^2)$ as in (3). The agent optimally chooses the action e^* reported in (6) and the coefficients of the linear payment scheme β^* and α^* are defined respectively by (7) and (8) with $k \equiv k(\sigma_*^2)$ and $\sigma^2 \equiv \sigma_*^2$.*

[3] As the first two component of (11) are positive and the third one is negative the first order condition can be satisfied for an appropriate $k(\sigma^2)$ function.

4 Agent's risk aversion and the provision of incentives

We now analyze how a variation in the agent's risk aversion affects the provision of incentives when, as in our framework, such a variation also induces a change in the technology adopted.

By applying the implicit function theorem to equation (11), we obtain that

$$\frac{\partial \sigma_*^2}{\partial r} = -\frac{\partial F/\partial r}{\partial F/\partial \sigma^2} = -\frac{-2rkk'\sigma^2 - k^2}{-4rkk' - k'' - 2rk'k'\sigma^2 - 2rkk''\sigma^2} < 0, \tag{13}$$

as the denominator is negative from the second order condition of maximization problem (9) and the numerator is also negative since the first order condition (11) implies that $-2rkk'\sigma^2 - k^2 = k'/r < 0$. This means that a reduction in the agent's risk aversion increases the riskiness σ_*^2 as well as the efficiency ($k(\sigma_*^2)$ goes down) of the technology chosen by the principal.

We will now show that while the reduction of the agent's risk aversion induces the principal to provide more incentives by increasing the agent's payment related to the output for any *given* technology (it is immediate from (7) that β^* is decreasing in r), this may no longer hold if the lower risk aversion of the agent leads the principal to change the technology employed (i.e., its efficiency and riskiness). In this case the characteristics of the new technology may affects the optimal provision of incentives in ways that counterbalance the former effect.

The total effect of a reduction of the agent's risk aversion on the optimal share β^* of output paid to the agent is obtained by total differentiation of (7) which gives

$$\frac{d\beta^*}{dr} = \underbrace{\underset{(-)}{\frac{\partial \beta^*}{\partial r}}}_{\textit{direct effect } (-)} + \underbrace{\underset{(-)}{\frac{\partial \beta^*}{\partial \sigma^2}}\underset{(-)}{\frac{\partial \sigma^2}{\partial r}}}_{\textit{indirect effect } (+)} + \underbrace{\underset{(-)}{\frac{\partial \beta^*}{\partial k}}\underset{(-)}{\frac{\partial k}{\partial \sigma^2}}\underset{(-)}{\frac{\partial \sigma^2}{\partial r}}}_{\textit{indirect effect } (-)}. \tag{14}$$

The first component in (14) represents the *direct effect* of a reduction of r on β^*, namely the effect on β^* if the same technology is employed. This component is equal to

$$\frac{\partial \beta^*}{\partial r} = -\frac{k\sigma^2}{(1 + rk\sigma^2)^2}, \tag{15}$$

and it is always negative as a lower risk aversion makes it optimal for the principal to give more incentives and less insurance to the agent, which requires increasing the payment related to output.

The other two components in (14) represent the *indirect effect* of the reduction of r on β^*, i.e. the effect caused by a change in the technology employed by the principal. The new technology is characterized by a higher efficiency and a higher riskiness which generate two opposing effects on β^*. The higher riskiness σ_*^2 of the project makes it optimal the provision of more insurance and less incentives to the agent, and this implies that the payment related to output decreases (we can call this the *riskiness effect*). Indeed, we obtain that

$$\frac{\partial \beta^*}{\partial \sigma^2} = -\frac{rk}{(1 + rk\sigma^2)^2} < 0. \tag{16}$$

On the other hand, the new technology is also characterized by a higher efficiency (i.e., a lower cost of effort k), which makes it optimal an increase of incentives as[4]

$$\frac{\partial \beta^*}{\partial k} = -\frac{r\sigma^2}{(1 + rk\sigma^2)^2} < 0. \tag{17}$$

This means that β^* increases as r goes down. We call this the *efficiency effect* and it goes in the same sign of the direct effect.

Therefore, the indirect effect due to the change of technology may in general lead to an increase or a decrease in β^*. We now try to understand under what conditions there is a definite sign in the relationship between r and β^*.

Let us first analyze the case where the indirect effect has the same sign of the direct effect, so that $d\beta^*/dr$ is always negative and, therefore, a lower agent's risk aversion leads to more incentives. From (14) it is immediate that this is the case when $(\partial\beta^*/\partial\sigma^2) + (\partial\beta^*/\partial k)k' \geq 0$ since $\partial\sigma^2/\partial r$ is always negative. Using (16) and (17), we obtain that this condition is satisfied when the elasticity $E_{k\sigma}$ of the technology with respect to the volatility is weakly greater than 1, i.e.,

Condition 1

$$E_{k\sigma} \equiv -k'\frac{\sigma^2}{k} \geq 1.$$

The intuition for this result is the following. If the function $k(\sigma^2)$ is elastic, then the increased efficiency of the technology (i.e., the reduction of k) associated to a given increase in its riskiness σ^2 is relatively large. This implies that the efficiency effect dominates the riskiness effect. Therefore, under Condition 1, the indirect effect has a negative sign and the reduction of the agent's risk aversion r always leads to an increase of β^*, which means that the principal will provide more incentives to the agent.

When $k(\sigma^2)$ is rigid and therefore Condition 1 does not hold, the efficiency effect is small relative to the riskiness effect and the indirect effect will be positive. As the direct effect has a negative sign, the total effect of a reduction in r on β^* will generally be ambiguous. However, if the increased riskiness of the new technology is sufficiently strong, then the lower agent's lower may induce a reduction of incentives.

The following proposition summarizes these results.

Proposition 2 *A reduction in the agent's risk aversion r generates two effects on the optimal share of output β^* paid to the agent. The direct effect always increases β^* while*

[4]This effect goes in the same direction of the direct effect generated by the reduction of r.

the indirect effect due to the change of technology can lead to an increase or a decrease of β^*. When Condition 1 is satisfied, both the direct and indirect effects have the same sign and a lower risk aversion r unambiguously increase β^* (i.e., $\partial\beta^*/\partial r < 0$). When Condition 1 does not hold, the total effect of r on β^* is generally ambiguous.

Let us now consider a specific functional form for the relationship between the cost parameter k of the agent's action and the variance of the shock σ^2. In particular, we assume that this technology function has a constant elasticity and it is $k = A(\sigma^2)^{-\eta}$, with $A > 0$, $\eta \in (0, 1/2)$ and $\sigma^2 \in (0, \infty)$ so that k is finite and positive for all σ^2. Then, $k' = -\eta k(\sigma^2)^{-1} < 0$ and $k'' = \eta(\eta+1)k(\sigma^2)^{-2} > 0$.

The first order condition (11) of the principal's maximization problem can be rewritten as

$$\eta \left(\sigma^2\right)^{\eta-1} + rA\left(2\eta - 1\right) = 0, \tag{18}$$

which implies that the variance of the optimal technology is equal to[5]

$$\sigma_*^2 = \left[\frac{\eta}{rA\left(1-2\eta\right)}\right]^{\frac{1}{1-\eta}}. \tag{19}$$

From $\eta < 1/2$ follows that Condition 1 is not satisfied (as $E_{k\sigma} = \eta < 1$) and the indirect effect is positive, i.e., the change of technology induced by the lower agent's risk aversion r leads to a reduction of β^* (the riskiness effect dominates the efficiency effect). This indirect effect opposes to the direct effect which instead pushes for an increase in β^*. The total effect of a reduction of r on β^* can be computed by substituting (15), (16), (17) and $\partial\sigma_*^2/\partial r$ (which is obtained from (19)) into (14). This leads to $\partial\beta^*/\partial r = 0$ which means that, in this special case, the direct and indirect effect of a change in r on β^* exactly offset each other and therefore that a reduction in the agent's risk aversion leaves the fraction of output paid to the agent unchanged.

5 Conclusions

We have shown that in a principal-agent model when the choice of the technology is endogenous for the principal the usual negative trade-off existing between the agent's risk aversion and the optimal incentive does not necessarily hold and can, in some cases, be reversed. This may occur when the link between the efficiency of the technology and its riskiness is weak. The reason is that for higher levels of agent's risk aversion the principal can decide to select less risky (and not much more inefficient) technologies that, in turn, make convenient the adoption of more powered incentive schemes.

[5] First note that $\eta < 1/2$ is necessary in order to get an interior solution. Second, it is immediate that $d\pi^*/d\sigma^2 \gtreqless 0$ if $\sigma^2 \lesseqgtr \sigma_*^2$. This means that profits are monotonically increasing in σ^2 when $\sigma^2 < \sigma_*^2$ and monotonically decreasing when $\sigma^2 > \sigma_*^2$, which confirms that profits are maximum at σ_*^2.

References

Aggarwal, R., Samwick, A., 1999. The other side of the trade-off: the impact of risk on executive compensation. Journal of Political Economy 107, 65–105.

Allen, D.W., Lueck, D., 1995. Risk preferences and the economics of contracts. American Economic Review 85, 447–451.

Cadsby, C.B., Song, F., Tapon, F., 2009. The impact of risk aversion and stress on the incentive effect of performance pay. Working Papers 0912, Department of Economics, University of Guelph.

Core, J., Guay, W., 2002. Estimating the value of employee stock option portfolios and their sensitivities to price and volatility. Journal of Accounting Research 40, 613–630.

Grund, C., Sliwka, D., 2010. Evidence on performance pay and risk aversion. Economics Letters 102, 8–11.

Holmstrom, B., Milgrom, P., 1987. Aggregation and linearity in the provision of intertemporal incentives. Econometrica 55, 303–328.

Legros, P., Newman, A.F., 2007. Beauty is a beast, frog is a prince: assortative matching with nontransferabilities. Econometrica 75 (4), 1073–1102.

Li, F., Ueda, M., 2009. Why do reputable agents work for safer firms? Finance Research Letters 6, 2–12.

Prendergast, C., 2002. The tenuous trade-off between risk and incentives. Journal of Political Economy 110, 1071–1102.

Rao, C., Hanumantha, H., 1971. Uncertainty, entrepreneurship, and sharecropping in India. Journal of Political Economy 79, 578–595.

Serfes, K., 2005. Risk sharing vs. incentives: contract design under two-sided heterogeneity. Economics Letters 88, 343–349.

Serfes, K., 2008. Endogenous matching in a market with heterogeneous principals and agents. International Journal of Game Theory 36, 587–619.

Wright, D.J., 2004. The risk and incentives trade-off in the presence of heterogeneous managers. Journal of Economics 83, 209–223.

Wulf, J., 2007. Authority, risk, and performance incentives: evidence from division manager positions inside firms. Journal of Industrial Economics 55, 169–196.