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"OPTIMAL POLLUTION CONTROL UNDER EMISSION CONSTRAINTS: SWITCHING BETWEEN REGIMES"

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Pollution Control under Emission Constraints: Switching between Regimes*

Enrico Saltari† and Giuseppe Travaglini‡§

Abstract

In this paper we develop a model of pollution control. The aim of the society is to maximize the net social benefits which are negatively affected by emission of pollutants. We show that the presence of an upper constraint on emissions affects pollution accumulation and welfare *even when* emissions are far below the level where the constraint binds. We get three main results. First, we find that for a society it is optimal to take into account the impact of the *latent* constraint at the outset of the planning horizon and to modify the emission flows even when the constraint is still slack. Second, we analyze how the restriction works effectively in the intermediate phases before the constraint binds. One crucial result is that the constraint on pollution growth may generate the unexpected result of *increasing*, instead of reducing, the emissions at the beginning and in the intermediate phases before the constraint binds. Finally, we state the boundary conditions to compute the optimal trajectory of emissions switching between regimes. This trajectory cannot be obtained by simply pasting together the unconstrained and the constrained parts of the trajectories.

Key words: Pollution control; inequality constraints; net social benefits.

JEL classification: E22, L51, H23, Q28.

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1 Introduction

In this paper we study the problem of pollution control in presence of emission constraints. To develop this idea, we assume that a regulator (e.g., a pollution control agency) sets the maximum growth rate of pollution which may be released in the environment in each period. We focus on two issues: (1) how much the emissions should be at each time to optimally control pollution accumulation and to maximize the net social benefits of a community, and (2) what is the optimal flow of emissions at each time, given an upper bound for the quantity of pollutants that can be released in the environment.

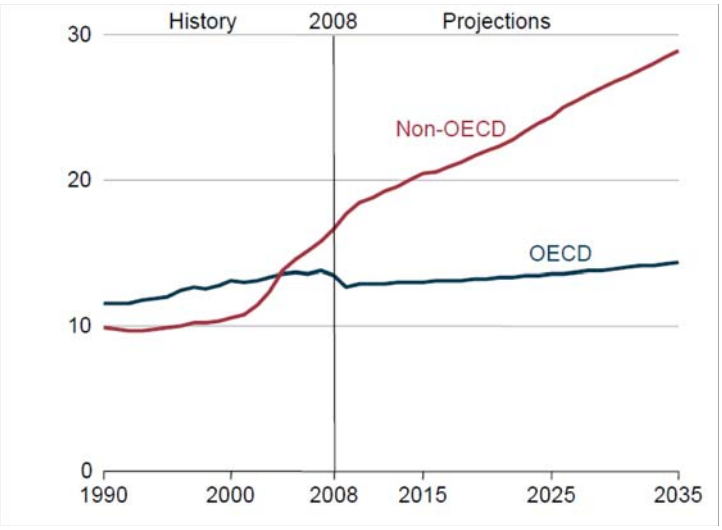
OECD provides annual evidence regarding emissions of pollutants in every region of the world. Data reveal some interesting trends about the recent evolution of the international economic-ecological system. Specifically, the emissions of long life pollutants, such as the greenhouse gas (GHG) emissions, are positively associated with economic growth, though there are some critical distributive aspects among regions. From the start of the industrial era, carbon dioxide emissions have increased by 30%, methane emissions by more than 100% and azote emissions by about 15%. As a consequence, in the latest decade the GHGs reached the highest concentration levels and growth rates ever compared with the atmosphere composition of the previous periods.

In addition, from the data it emerges that in the next future the macro-regions of the world that are major contributors to the growth of GHG are subject to change. Non-OECD countries like Brazil, China and India will contribute strongly to the growth rates of emissions, whereas the OECD countries are going to reduce their rates of emissions. Figure 1 shows the projections for the carbon dioxide estimated by the IEO (International Energy Outlook, 2011). In the next decades, after the crude acceleration started in 2004, energy-related emissions of CO_2 will increase faster in the Non-OECD countries than in the OECD countries.[1]

Projections are, to the extent possible, based on existing environmental laws and regulations. Thus, projections for emissions may change significantly if laws and regulations aimed at reducing emissions are changed or new ones are introduced. Many countries have submitted emission reduction goals to environmental regulation. The Kyoto protocol (1997), the Copen-

[1]Energy-related carbon dioxide emissions are defined as emissions related to the combustion of fossil fuels (liquid fuels, natural gas, and coal) and those associated with petroleum feedstocks. Emissions from the flaring of natural gas are not included.

Figure 1 World energy-related emissions of CO_2, 1990-2035 (billion metric tons). Source: IEO.



hagen (2009) negotiations and the Cancun Conference (2010) are the most prominent examples of multilateral agreements aimed at controlling pollution growth. The emissions mitigation pledges fall into two general categories: absolute reductions and intensity reductions. Absolute reductions reduce GHG emissions independent of economic or material output. The European Union, the United States, Japan, Russia, and Brazil have announced absolute reduction goals expressed as percentage reductions below historical base-year amounts. China and India have announced intensity reduction goals, which typically are expressed as reductions in emissions per unit of output measured by Gdp.

Also, environmental regulation is used by national governments to attain pollution targets in order to defend the quality of environment and the human health in a country. However, there is a deep debate about the ability of these rules to affect pollution trends over time (Tol, 2008, 2012; Carraro and Siniscalco, 1993). One of the main issue concerns the conditions which must prevail to make the environmental rules credible. To say, it is stressed that since the costs of emission reduction vary widely among countries, and since unilateral reduction may be too expensive for a single country in terms of aggregate output (Hoel, 1991), the targets on emissions risk to be neither

credible nor enforceable.

Others similar considerations do not exhaust all the critical aspects related to the implementation of environmental regulations (Baumol and Oates, 1989; Perman et al., 2003; Tirole, 2012). Rather, some aspects remain unexplored. As far as we know, little research has been done on how such rules, either national or multilateral, would operate in practice when they are credible, and when the environmental regulations on emission trends operate over a *limited* planning horizon, which is the standard case (i.e. Kyoto protocol). Therefore, in what follows we will focus on this issue studying the effectiveness of credible regulations over a time horizon exogenously specified.

Said that, a huge amount of literature on pollution control exists. As it is well known, in static models the problem of pollution control is straightforward. Pollution is treated as a negative externality, and emissions taxes and abatement subsidies are used to attain the efficient level of pollution stock (Pigou, 1920; Baumol and Oates 1989; Fisher, 1981; Pearce and Turner, 1990; Kolstad, 2000; Perman et al., 2003).

Less obvious is, instead, the problem of pollution control in a dynamic setting with and without environmental constraints on emissions (Smith, 1972; Plourde, 1972; Forster 1973; Siebert, 1987; Bromley, 1995; Van der Ploeg and Withagen, 1991; Tahvonen, 1995**;** Perman et al., 2003; Saltari and Travaglini, 2011a, 2011b). Traditionally, these kind of environmental problems are analyzed considering the *binary* case of constrained versus unconstrained regime (Fisher, Kutrilla and Cicchetti, 1972; Dasgupta and Heal, 1979; Fisher, 1981; Hartwick and Olewiler, 1986; Baumol and Oates, 1989; Boyer and Laffont 1999). However, this approach – formally based on the Kuhn-Tucker conditions – often provides little insights into the actual problem of pollution control over time. In fact, the binary approach does not allow to get answers to the following questions: Is the optimal emission policy of a society affected in the *intermediate* phases before the constraint binds? Do forward-looking agents – consumers, firms and society – *anticipate* the effects of environmental constraints in planning their *current* economic-ecological decisions? Can the imposition of *latent,* i.e. not yet binding, constraints affect the emission flows and the accumulation of pollution even when the constraint is still slack? These are the questions addressed in this paper.

To develop this idea, we assume that the aim of the society is to maximize the net social benefits which depend on aggregate output and pollution. Regrettably, the relationship between output and pollution is often a trade-off between development and preservation. Therefore, we model this fact assum-

ing that pollution is a by-product of economic activity, and that polluting emissions affect negatively net benefits. Given the trade-off, society controls emission flows but any variation of emissions affects not only the stock of pollution over time, but also the amount of goods and services available to the community in each period.

Intertemporal features are a basic element in this framework. Importantly, we are primarily concerned with those scenarios where polluting emissions are unconstrained at the current time (the constraint is slack), but they will be constrained at some future date (the constraint binds). Our aim is to focus on the intertemporal conditions which drive the economic-ecological system through these two alternative regimes. Our problem is related to the literature on optimal stopping in a context where the evolution of the state variable, the pollution stock, is not exogenous (Dixit and Pindyck, 1994).

We get three main results. (1) We find that, given the regulation, it is optimal for a society to anticipate the impact of the latent constraint and to modify the flow of emissions *before* the constraint binds. (2) We compare the unconstrained and constrained scenarios to explain how the restriction works effectively in the intermediate phases when the restriction is still slack. Precisely, we show that the effectiveness of regulatory policies depends on the starting conditions of the system and the time available to implement a policy. Our framework generates some counter-intuitive paths. We provide examples of how future constraints on emissions tend to rise, instead of decreasing, the rate of emissions in the intermediate phases before the constraint binds. (3) Finally, we state the boundary conditions necessary to compute the optimal trajectory switching between regimes. We show that the switching trajectory cannot be obtained by simply pasting together the unconstrained and the constrained parts of the trajectories in the two alternative regimes.

It is remarkable that in our model the environmental constraint on pollution growth has, under certain conditions, unexpected effects on the economic-ecological decisions of the society. Since pollution is a by-product of economic activity, the society can try to compensate for lower pollution accumulation when the constraint is active, raising emissions (economic activity) before that time. Therefore, future limitations can have a positive or negative impact on both current pollution growth and welfare, depending on the economic-ecological conditions prevailing when the regulatory policy is implemented.

The paper is organized as follows. In sections 2 and 3 we develop a model for emissions control in order to maximize net social benefits. Next,

in sections 4 and 5 we apply the model to the unconstrained and constrained regimes. We scrutinize how the latent constraint on pollution accumulation affects the control problem at the current time and the optimal pollution path selected to maximize net social benefits. Further, in section 5 we formalize the boundary conditions necessary to calculate the trajectories switching between regimes. Section 6 concludes.

2 A pollution control model

Our aim at this stage is to formulate a model for guiding choices between social benefits and costs. The model presented here is a special version of Fisher, Kutrilla and Cicchetti (1972), Kamien and Schwartz (1991), Kolstad and Krautkraemer (1993), Tahvonen and Salo (1995) and Saltari and Travaglini (2013).

It starts from the assumption that gross benefits depend positively on economic activity, but that pollution is a by-product of economic activity. Therefore, we propose the maximization of the following objective function:

$$NSB = \int_0^T [b(p_t) - d(z_t)]\, e^{-\rho t} dt \tag{1}$$

where $b(p_t) - d(z_t)$ are the net benefits at time t derived by p_t units of pollution stock and z_t units of polluting emissions caused by production. ρ is the constant social discount rate. The interval $[0, T]$ defines the planning horizon.

In what sense is pollution p_t beneficial? One answer comes from the fact that producing some goods or services, that we do find useful, may not be possible without generating some pollution, even if in a limited amount. Therefore, the larger is the economic activity, the higher is the amount of goods and services available to the community, and the greater will be the stock of pollution p_t that the community is willing to undergo.

To get a gross benefit function as $b(p_t)$ let's indicate with k_t the amount of a dirty input (i.e. oil) employed in production. Then, assume that the function $g(k_t)$ gives the instantaneous gross social benefits in terms of goods from any economic activity that uses k_t. A type of benefit function is

$$g\left(k_t\right) = k_t^{\alpha} \tag{2}$$

where $0 < \alpha < 1$. To incorporate pollution p_t into the function (2), we model p_t as a by-product of k_t. A basic specification can be

$$p(k_t) = \frac{c}{A} k_t^{\beta} \tag{3}$$

where c is the parameter which measures the marginal propensity to pollute of the community, and $\beta \gtreqless 1$ is the elasticity degree (Smulders and Gradus 1996). A is the parameter which captures the effect of abatement. We assume a negative link from abatement to pollution accumulation. Substituting by k_t in equation (2), we can re-write the gross benefit function in terms of pollution, that is

$$b\left(p_t\right) = h p_t^{\theta} \tag{4}$$

where $h = \frac{A}{c}^{\alpha/\beta} > 0$ and $0 < \theta \equiv \frac{\alpha}{\beta} < 1$ by assumption. In other words, we assume that the marginal social benefits are a decreasing function of p_t. Therefore, $h p_t^{\theta}$ provides the gross social benefits from any economic activity that "generates" pollution p_t.

2.1 The social damage

Anyway, polluting emissions have a detrimental effect on environment and human health. Hence, we have to quantify their underlying social damages. Let's indicate with z_t the emission flows, and assume that this (control) variable is in the hands of the society. Function $d(z_t)$ is the social damage which depends on z_t. It has the form

$$d(z_t) = z_t + \frac{\omega}{2} z_t^2 \tag{5}$$

Equation (5) states that the social damage is caused by the emission z_t in a non linear manner. $d(z_t)$ includes the additional increasing quadratic term $\frac{\omega}{2} z_t^2$. It captures the idea that additional units of emissions increase more than proportionally the disutility endured by society: an acceleration of the rate of emissions z_t increases the social costs of any incremental unit of pollutant released in environment. In other words, for a society emissions are costly in term of welfare, and the marginal adjustment cost is increasing in the size of emissions. In short, the social *net* benefits can be re-written as

$$b(p_t) - d(z_t) = h p_t^{\theta} - z_t - \frac{\omega}{2} z_t^2 \tag{6}$$

To close the model, we specify the net rate of pollution accumulation $\dot{p} \equiv \frac{dp}{dt}$ as the difference between the emissions z_t and the ecological decay δ of the pollution stock. Pollution decay is assumed to be a linear function of the pollution stock level p_t. Hence, the stock of pollution has a dynamics represented by the equation of motion

$$\dot{p}_t = z_t - \delta p_t \tag{7}$$

where the net accumulation of pollution is equal to zero ($\dot{p}_t = 0$) when $z_t = \delta p_t$.

3 The optimal control problem

We have now enough structure to study our general problem (1). The objective is to choose a sequence of emissions z_t to maximize net benefits, that is the difference between *all* the benefits and *all* the damages of pollution. We can write

$$\max_{z_t} \; NSB = \int_0^T \left(h p_t^{\theta} - z_t - \frac{\omega}{2} z_t^2 \right) e^{-\rho t} dt \tag{8}$$

subject to

$$\dot{p}_t = z_t - \delta p_t \tag{9}$$

$$p(0) = p_0 \tag{10}$$

$$p(T) = p_T \tag{11}$$

The interval $[0, T]$ represents the planning horizon, with p_0 and p_T the stocks of pollution at the beginning and at the end of the period.

It is conceivable that the set of values that pollution can take on is restricted because of environmental regulation. Let us explore the case where the growth rate of pollution cannot exceed a given upper bound imposed by an environmental protection agency. Precisely, the growth rate of pollution $\dot{p}$ cannot exceed an upper bound regulated by environmental control laws. The target is credible by assumption. We assume that the dynamic inequality constraint takes the form

$$\dot{p}_t \leq x p_t \tag{12}$$

The proportion $x < 1$ defines the upper bound on the pollution growth rate in each period, given the stock of pollution p_t at that time. Obviously, since the pollution rate $\dot{p}_t$ depends on the control variable z_t, the upper bound (12) implicitly imposes restrictions on emissions z_t. Therefore, the constraint (12) influences the emissions policy at all times over the period subject to optimization.

The Hamiltonian for the problem (8) is

$$H(p_t, z_t, \mu_t) = \left[h p_t^{\theta} - z_t - \frac{\omega}{2} z_t^2 + \mu_t \left(z_t - \delta p_t \right) \right] e^{-\rho t} \tag{13}$$

where μ_t is the costate variable of the pollution stock p_t. The associated Lagrangian is

$$\mathcal{L}(p_t, z_t, \mu_t, \lambda) = H(p_t, z_t, \mu_t) + \lambda_t \left(x p_t - z_t - \delta p_t \right) e^{-\rho t} \tag{14}$$

where the Lagrangian multiplier λ_t is the shadow value of the upper bound $x p_t - \dot{p}_t \geq 0$.

Ignoring time subscripts to simplify notation, the optimal solution must satisfy the following conditions:

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial z} &= 0 : -(1 + \omega z) + \mu - \lambda = 0 && (15) \\ \text{with } \lambda &\geq 0, \quad x p - \dot{p} \geq 0 \quad \text{and} \quad \lambda \left(x p - \dot{p} \right) = 0 && (16) \end{aligned}$$

$$-\frac{\partial \mathcal{L}}{\partial p} + \rho \mu = \dot{\mu} : -h \theta p^{\theta - 1} + (\rho + \delta) \mu - \lambda (x - \delta) = \dot{\mu} \tag{17}$$

$$\frac{\partial \mathcal{L}}{\partial \mu} = \dot{p} : z - \delta p = \dot{p} \tag{18}$$

4 The unconstrained regime

Let's assume that no regulation exists, so that no restrictions are placed on pollution growth. In this scenario we can use the standard maximum principle to solve the problem (8) since $\lambda = 0$. In this regime, the optimal level of emission z is given by condition (15)

$$z = \frac{1}{\omega} (\mu - 1) \tag{19}$$

which states that for a society is optimal to increase emissions, and production as well, when $\mu > 1$. Notice that for a society μ is the shadow value of the emissions z. Therefore, to maximize the functional NSB it is convenient for a society to rise z when the shadow value μ is larger than the marginal social cost equal to 1. Reducing emission is optimal in the opposite case.

To complete the analysis, let's focus on equation (17) when $\lambda = 0$. It describes how the imputed worth μ of one unit of pollution moves along the efficient path at any instant t. The costate variable μ equals the present value of marginal benefits $h\theta p^{\theta-1}$ arising from the impact of a marginal unit of pollution *plus* the present value of its variation $\dot{\mu}$. The shadow variable μ represents the opportunity value of intended goods and services, since it quantifies the change in the net benefits caused by an additional unit of pollution. Dividing both sides of (17) by μ, we get

$$\rho + \delta = \frac{h\theta p^{\theta-1}}{\mu} + \frac{\dot{\mu}}{\mu} \tag{20}$$

This equation states that along the optimal pollution path the value of the social benefit of pollution $\frac{h\theta p^{\theta-1}}{\mu} + \frac{\dot{\mu}}{\mu}$ must be equal to its social user cost, $\rho + \delta$. Thus, when a unit of pollution is increased at time t, its user cost is $\rho + \delta$, but its marginal benefit is $h\theta p^{\theta-1}$. This implies that, since $\rho + \delta > 0$, the sign of the shadow price μ must be positive, $\mu > 0$.

To shed light on the interdependencies in the ecological-economic system, it will be convenient to rearrange the canonical equation (20) as $\mu = \frac{h\theta p^{\theta-1} + \dot{\mu}}{\rho+\delta}$, and then to substitute it in (19) to get:

$$z = \frac{1}{\omega}\left[\frac{h\theta p^{\theta-1} + \dot{\mu}}{\rho + \delta} - 1\right] \tag{21}$$

This expression states that society aims at maximizing the net social benefits by selecting, right from the start, the optimal emissions z which satisfy the first order condition of the problem (8). This trajectory for z is calculated considering not only the present value of marginal utility $h\theta p^{\theta-1}$ at time t, but also the change in benefits $\dot{\mu}$ (costs) in future periods. Therefore, knowledge of μ and of its expected change $\dot{\mu}$ are necessary conditions to sort out the optimal path of z which maximizes welfare, given the time horizon. But, the following question arises: given this expectation, how do future restrictions on pollution growth affect the *intermediate* course of emissions and welfare before the constraint (12) binds?

4.1 Phase diagram analysis of optimal unconstrained pollution control

By now, let's continue our analysis of the unconstrained regime. By differentiating the first order condition (19) with respect to time, we get the following differential equation in z:

$$\dot{z} = \frac{1}{\omega}\dot{\mu} \tag{22}$$

Substituting by μ and $\dot{\mu}$ in equation (17) and rearranging, we obtain the dynamics of z along the optimal path. With this transformation we get an intertemporal problem expressed in term of the *control-state* variables rather than *state-costate* variables. The system of differential equations is given below

$$\dot{z} = (\rho+\delta)z + \frac{1}{\omega}\left(\rho+\delta-h\theta p^{\theta-1}\right) \tag{23}$$

$$\dot{p} = z - \delta p \tag{24}$$

which describes the unconstrained evolution of z and p over time.

Our first task is to determine the locus of the points in the (z,p) plane along which $\dot{p}=0$. From (24) $\dot{p}=0$ if $p^* = \frac{z}{\delta}$. This curve is drawn in figure 2. It is linear and increasing and goes trough the origin.

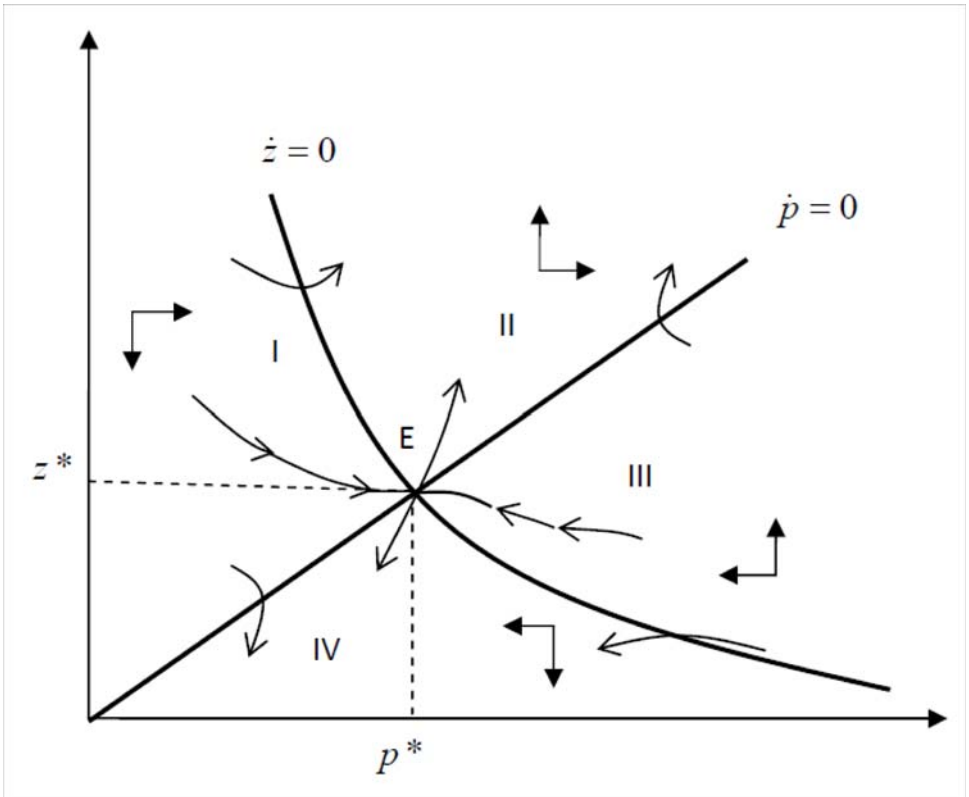
Next, we turn to the $\dot{z}=0$ locus. From (23), $\dot{z}=0$ if

$$z^* = \frac{1}{\omega}\left(\frac{h\theta p^{\theta-1}}{\rho+\delta} - 1\right) \tag{25}$$

where $h\theta p^{\theta-1} > 0$ by assumption. Notice that by condition (22) $\dot{z}=0$ implies $\dot{\mu}=0$, so that from (20) we can write $\mu^* = \frac{h\theta p^{\theta-1}}{\rho+\delta}$. To draw the isocline $\dot{z}=0$ in figure (2), recall that $\theta<1$, that is the marginal benefit is decreasing in p and the locus $\dot{z}=0$ has a negative slope. This assumption has a good economic and ecological sense.

The loci $\dot{p}=0$ and $\dot{z}=0$ define four regions labeled I to IV in figure 2. Above the $\dot{p}=0$ locus $z>\delta p$, which implies that $\dot{p}>0$. Similarly, below the locus $\dot{p}=0$ the sign of the variation is negative, $\dot{p}<0$, since $z<\delta p$, and the stock of pollution decreases. Hence, going from east to west in figure (2) the sign sequence of $\dot{p}$ is $(+,0,-)$. The arrows in the phase diagram summarize this information.

Figure 2 Optimal pollution stock



Accordingly, from (23) and (25) it is clear that, for a given p, above the $\dot{z}=0$ locus $\mu \equiv \frac{h\theta p^{\theta-1}}{(\rho+\delta)} > 1$, implying $\dot{z} > 0$. Then, below the locus $\dot{z}=0$ the sign of the variation is negative, $\dot{z} < 0$, since $\mu < 1$, and the z decreases. Therefore, going from south to north in figure (2) the sign sequence of $\dot{z}$ is $(-, 0, +)$. The arrows summarize this information.

Figure 2 combines the two loci. The diagram shows how z and p must behave to satisfy the system (23)-(24). All trajectories in region I point southeast. They can enter in region IV only by crossing the $\dot{p}=0$ locus, and the slope of this trajectory is infinite at the crossing, since it is given by $dz/dp = dz/dt/dp/dt = \dot{z}/\dot{p}$ and the denominator is zero on the locus $\dot{p}=0$. On the other hand, when a trajectory goes through the locus $\dot{z}=0$, its slope is zero. Hence, if a trajectory in region I reaches the locus $\dot{z}=0$ it turns up. Since the trajectories cover the whole space, there is one trajectory in region

I that reaches the equilibrium point E. We refer to it as the stable path.

Similar reasoning reveals the existence of an upward stable path in region III, while regions II and IV each possess an unstable path leading away from E, identifying E as a saddle point. The point E identifies a sustainable equilibrium since the pollution stock p (and the underlying capital stock k) reaches a stable value in the long-run.

Our diagrammatic characterization of the equilibrium E as a saddle point can be confirmed locally by linearizing the differential equations (23)-(24) around E. The matrix Ω of coefficients of the linearized system is

$$\Omega = \begin{bmatrix} \rho + \delta & -\frac{1}{\omega} h\theta \left(\theta - 1\right) \left(p^*\right)^{\theta - 2} \\ 1 & -\delta \end{bmatrix} \tag{26}$$

confirming that the interior equilibrium is a saddle point since Ω, evaluated at $\left(p^*, z^*\right)$, has a negative determinant.

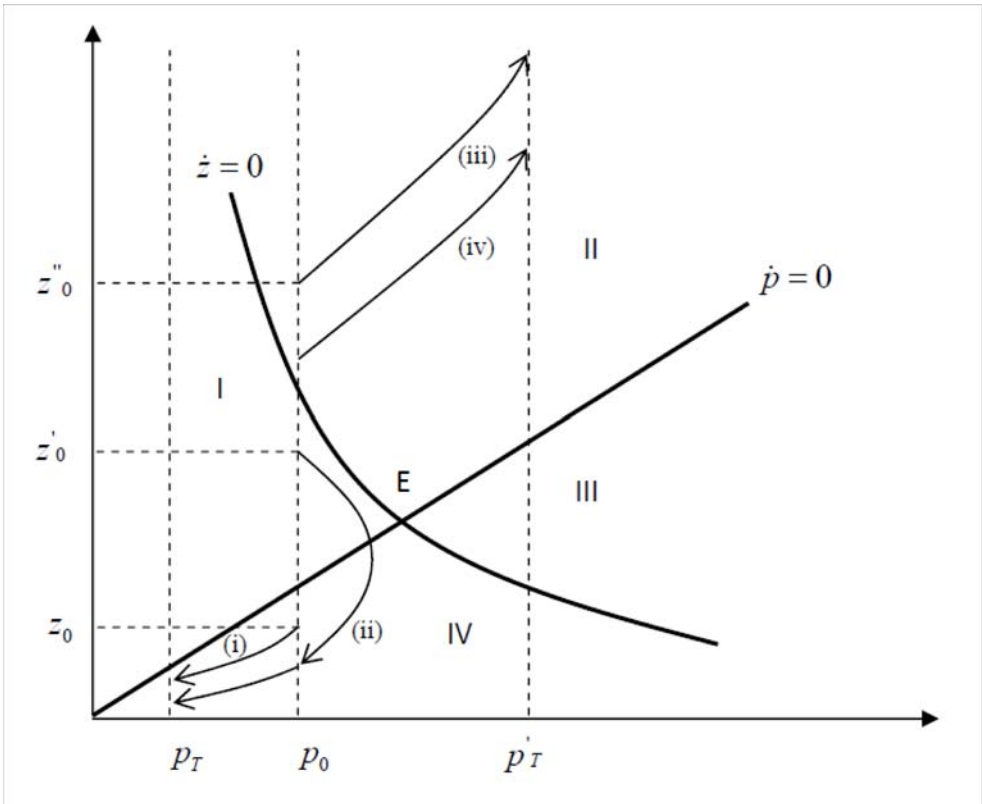
4.1.1 Environmental policy

How does environmental policy work in the unconstrained regime? As said above, the boundary values (10) and (11) state that a society inherits a pollution stock p_0 from the past and that its aim is to reach p_T units of pollution at time T. The value of T and the boundary values of p_0 and p_T are specified exogenously. For instance, an environmental rule can aim at reducing the pollution stock over the time interval $[0, T]$ in order to get $p_T < p_0$. Basically, the society controls z and chooses the optimal trajectory which maximizes the net social benefits. Some admissible *unconstrained* pollution paths are drawn in figure (3).

The vertical dashed lines through p_0 and p_T show that, although the boundary values for the state variable are exogenously given, it is possible to select the optimal values for the control variable z. In practice, the environmental policy depends both on the objectives and the amount of time T available to the society. The shorter the time available, the higher (positive or negative) will be the required rate of pollution emission z, and the greater the distance between the selected trajectory and the $\dot{z} = 0$ locus.

For a practical policy in the unconstrained regime, no matter how large the value of T, it is always possible to select an optimal time path which satisfies conditions (10) and (11). If the society has a lot of time available, it can choose a trajectory that comes arbitrarily close to E, where the dynamics

Figure 3 Some unconstrained paths



of the control and state variables becomes very slow. When, on the other hand, time is short, the only way to achieve p_T at time T is to select a higher trajectory. Let's discuss three basic cases.

Region IV depicts the paths to an "ecological paradise". Assume that for an initial p_0 the society will adopt an emission rate z_0 which lies below the $\dot{z} = 0$ locus, such as the path (i) which allows to reach p_T at time T. Then, the pollution stock will be reduced – but also the use of the dirty input, and the corresponding level of the economic activity – since society operates to decrease the emissions z until pollution stock contracts to p_T. However, if T is larger the society may prefer a different environmental policy. This is described by the optimal path (ii). This path begins in region I at $z_0^{'}$ where society finds initially optimal to *rise* the pollution stock, ensuring however that the boundary value p_T is reached at time T. In other words, if there is enough time it will be optimal for maximizing the net social benefits to increase the use of (polluting) input at the beginning, rising in the intermediate phases the pollution stock, so that p goes through a peak.

Other boundary values would yield different paths. Suppose that the initial pollution stock is still p_0, but the terminal stock is now $p_T^{'} > p_0$. The trajectories (iii) and (iv) depict two alternative paths in the "development versus preservation" scenario. Indeed, along these paths z and p always increase and the use of the dirty input k and the corresponding level of economic activity (which is in the background of the model) increases as well. It is remarkable that in region II all trajectories sum up the trade-off between economic development and environmental quality. This feature is typical for the new industrialized countries, like China and India, which prefer to increase the use of dirty inputs, and the output per person, rather than to improve the environmental quality. Thus, compared to the previous scenario, the paths in region II describe the dynamics of a system shifting towards an "ecological hell".

Finally, notice that the paths in region II can also imply a complete "environmental collapse" when the boundary values impose that $p_T < p_0$ (Siebert, 1992). If at the initial time the society chooses a level of z_0 which lies above the $\dot{z} = 0$ locus – or correspondingly chooses a too low emission tax rate $-$ μ – then emission flows rise steadily over time and the stock of pollution increases. Therefore, if the initial level of z_0 lies in region II, but the boundary values state $p_T < p_0$, there is no feasible solution to the pollution control problem because in this region all trajectories go up towards the right.

5 Towards the constrained regime

Let us now study the transition from the unconstrained to the constrained regime. Following our intuition, we cannot assume that $\lambda > 0$ along the entire planning horizon $[0, T]$. Indeed, if this condition holds in all periods the accumulation rate of pollution rises steadily at its maximum constrained rate $\dot{p} = xp$. In other words, if $\lambda > 0$ at $t = 0$, the constraint (12) binds at the starting time and for the time to come, and the emission z is at its maximum level in each period

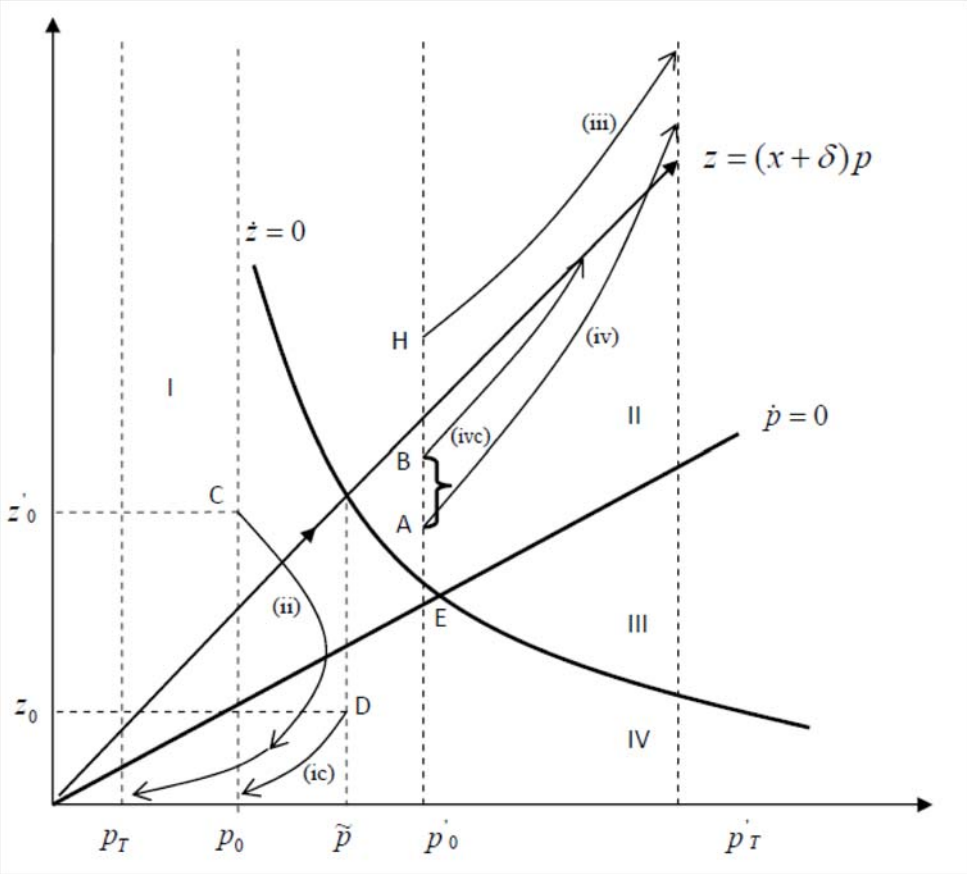
$$z_t = (x + \delta)p_t \tag{27}$$

This is the equation of the boundary curve. An important remark is the following. In figure 4, the boundary curve (27) lies above the locus $\dot{p} = 0$ since along this isocline $z = \delta p$. Therefore, in the constrained regime the set of admissible paths and controls lies in the space defined by the boundary curve $z = (x + \delta)p$ and the p axis. Note that when x tends to infinity the boundary curve coincides with the vertical axis, meaning that the dynamics of the system is unconstrained, whereas the same equation (27) coincides with the locus $\dot{p} = 0$ when $x = 0$.

Thus, to study the transition of the system from one regime to another we must assume that the constraint (27) is slack at time $t = 0$, but that it will be binding at some future date $t^* < T$. As said above, in this scenario the Kuhn-Tucker conditions are not suitable to describe the complexity of the problem. Therefore, our aim is to identify the necessary and sufficient conditions which drive the free trajectories towards the boundary curve (27) in the interval $[0, T]$.

To begin with, look at figure 4. Let $\tilde{p}$ denote the value of the pollution at the intersection between the $\dot{z} = 0$ locus and the boundary curve (27). Starting from a point along the boundary curve, but smaller than $\tilde{p}$, the trajectory moves upwards along the curve. Given that in the proximity of this bound the trajectories can enter in region I, where the constraint ceases to be active, they could, on occasions, stray away from the boundary curve: once in region I, trajectories can diverge from the constraint moving down on the left. On the other hand, for values larger than $\tilde{p}$ the trajectory cannot leave the boundary curve because in region II the emission z can only increase. Therefore, in this region if z diverges from the upper bound, countervailing forces would tend to bring it back to its constrained path. This is displayed in figure 4.

Figure 4 Some constrained paths



As for the unconstrained regime, the choice of the optimal path of emissions depends on the values p_0 and $p_{T,}$ and the time T. Here, however, we have to respect the constraint (27) which also implies that $\lambda > 0$ when it binds. This gives us the chance to compare the optimal trajectories with and without the constraint. This is what we do in figure 4 which is to be compared with figure 3.

First of all, observe that if $p_T < p_0$ optimal trajectories cannot start above the boundary curve in region I. Thus, in this scenario the feasible trajectories which drive the system towards the "ecological paradise" are restricted as well. In other words, not all the trajectories are admissible even when the aim is to reduce the pollution stock. Indeed, the condition (27) states that the trajectory (ii), starting in point C, is no longer admissible. In addition, note that when the control z crosses the boundary curve it starts immediately to move along it in the northeast direction. But, when $p_T < p_0$ the trajectories in region IV are still admissible since they lie in the unconstrained space below the boundary curve. Therefore, the optimal *unconstrained trajectories in the constrained regime*, if they exist given time T, must lie below the boundary curve and the current emission z must be reduced quite a lot at the outset to maximize the net social benefits of problem (8). The trajectory (ic), which starts from point D in figure 4, is an example of admissible trajectory in the constrained scenario driving towards the "ecological paradise".

Similar problems arise in the opposite scenario of "ecological hell". To study how trajectories behave in region II, let's assume different boundary values for both the initial and final pollution stock with $p_T^{'} > p_0^{'}$. In this scenario, the unconstrained path (iii) starting from point H is no longer admissible. Further, if time T is short there may not be enough time to move from $p_0^{'}$ to $p_T^{'}$ following the unconstrained path (iv), which starts in point A, given the maximum growth rate allowed by the constraint $\dot{p} = xp$. Thus, if the initial level of emission lies in region II the only way to achieve p_T at time T is *to increase* the emission z – that is to rise the economic activity and the use of dirty inputs – *jumping* at a *higher* level of z at the outset, and moving along the trajectory (ivc) which begins at the point B. The distance from A to B in the figure represents the graphical counterpart of the jump of z at the starting period. In fact, when the constraint (27) is binding, the pollution stock increases more slowly than along the unconstrained path (iv). In other words, to achieve p_T in T, society must be willing to initially bear a level of emission z *higher* than the one of the unconstrained regime so as to meet the boundary curve at the optimal time t^* which allows to get p_T at T.

The main implication of this forward-looking behavior is that the society finds optimal to modify its emission flows at the beginning and even in the intermediate phases when the constraint is still slack, planning to meet the boundary curve at some future (endogenous) time t^*. A crucial consequence of this anticipative behavior is that the restriction of pollution growth may generate the unexpected result of *raising* the emissions in the short-run and in the intermediate periods before the constraint binds. Only in the long-run the cut of emissions is obtained.

There is an additional consequence related to this outcome. The constraint on emissions affects the pollution accumulation when it is far away from the boundary curve: not only it is optimal to begin with a higher level of emission, but it is also optimal to change the trajectory in the intermediate phases when the constraint is *still* slack. The imposition of the constraint (27), thus, has rather far-reaching effects on the selected path of emissions.

On the practical ground this means that environmental regulation, when it is credible, may affect emission flows even when the restrictions are slack at the current time. This is a good news. However, the bad news is that, given the trade-off between economic development and environmental quality, the impact of the regulatory constraint on pollution growth may be different from what expected, even when countries pledge to respect the environmental commitment. In this perspective, the main implication of our analysis is that future limitations on emissions can have a positive or negative impact on pollution growth at the current time, depending on the economic-ecological conditions prevailing at the time when the environmental constraint is announced.

Therefore, the emission policy for communities (i.e. countries) adopting the constraint will not generally be the same as for communities that will never adopt the constraint. Indeed, compared to the unconstrained scenario, the potentially constrained community finds optimal to modify its emission flows even when they are far below the upper bound. This may sound complicated but it is the natural consequence of the intertemporal optimization of net social benefits.

5.1 The analytical solution

The previous considerations have the analytical counterpart in the method employed to compute the switching trajectories. In our model, over the planning horizon $[0, T]$ the optimal emission policy must ensure that the

path for p (and its shadow value μ) is *continuous and smooth* in any period t. In other words, in the constrained scenario the dynamics of the system must be such that when the pollution path meets with the boundary curve $(x+\delta)p$ at some time t^*, with $0 < t^* \leq T$, the variable p converges to the constrained status smoothly and without discontinuity. In fact, if the path for p were discontinuous it would be impossible to compute its first derivative and this would violate our requirement that the slope of the two trajectories must be equal (smoothness) at the contact point at time t^*. This is the crucial implication of our analytical solution: latent constraints exert immediate effects on current decisions, and any switching trajectory of p_t is a new path, continuous and smooth, along the entire planning horizon.

This intuition has a formal counterpart.

In analyzing the path from time 0 to time T, we begin by considering the unconstrained system of equations (23)-(24) in its linearized version whose Jacobian is given by the matrix Ω (see equation (26)). Let $\varepsilon_1 > 0$ and $\varepsilon_2 < 0$ be the characteristic roots associated with Ω, and $\begin{pmatrix} B_1 & C_1 \end{pmatrix}$ and $\begin{pmatrix} B_2 & C_2 \end{pmatrix}$ the corresponding eigenvectors.[2] The general solution of this system has the form

$$z_t = d_1 B_1 e^{\varepsilon_1 t} + d_2 B_2 e^{\varepsilon_2 t} + z^* \tag{28}$$

$$p_t = d_1 C_1 e^{\varepsilon_1 t} + d_2 C_2 e^{\varepsilon_2 t} + p^* \tag{29}$$

where d_1 and d_2 are constants determined by the appropriate boundary conditions, and z^* and p^* are the steady state values.

In the *unconstrained* regime the boundary conditions allow to compute the value of d_1 and d_2 which provide the optimal trajectory from p_0 to p_T of both the state and control variables p and z. Therefore, using the boundaries (10) and (11) we require that at $t = 0$ the pollution stock is p_0, and that at T the pollution stock is p_T:

$$p_0 = d_1 C_1 + d_2 C_2 + p^* \tag{30}$$

$$p_T = d_1 C_1 e^{\varepsilon_1 T} + d_2 C_2 e^{\varepsilon_2 T} + p^* \tag{31}$$

[2] We normalize the eigenvectors by imposing the restriction

$$B_i^2 + C_i^2 = 1, \; i = 1, 2$$

Hence, we have two equations to compute the values of d_1 and d_2, say d_1^* and d_2^*, which satisfy the system over the interval $[0, T]$.

Can these values d_1^* and d_2^* satisfy the system of equations in the potentially constrained regime?

As explained above, the boundary curve (27) modifies the optimal trajectory even in the intermediate phases when the constraint is still slack since the control set in now limited. This implies that the value of the constants which satisfy the system of equations switching between regimes must be different from d_1^* and d_2^*. As a result, we need some additional boundary conditions to tie down their values and to compute the optimal time t^* when the unconstrained and constrained paths meet each others. In other words, since in the constrained plane (p, z) the optimal trajectory cannot begin anywhere along the $p = p_0$ line and cannot end anywhere on the $p = p_T$, the starting point of the switching path will change at the beginning to *anticipate* the effect of the boundary curve when it binds (27).

To assure optimality along the switching path we impose that it satisfies the following conditions:

$$d_1 C_1 + d_2 C_2 + p^* = p_0 \tag{32}$$

$$\dot{p}_{t^*} = x p_T e^{x(t^*-T)} \tag{33}$$

$$\int_0^{t^*} \dot{p}_t dt + \int_{t^*}^{T} x p_T e^{x(t-T)} dt = p_T - p_0 \tag{34}$$

where:

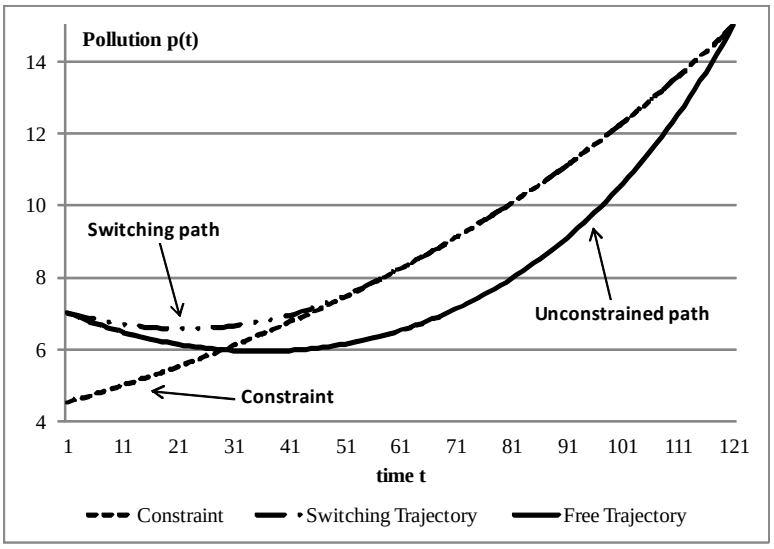
$$\dot{p}_{t^*} \equiv d_1 B_1 e^{\varepsilon_1 t} + d_2 B_2 e^{\varepsilon_2 t} + z^* - \delta \left(d_1 C_1 e^{\varepsilon_1 t} + d_2 C_2 e^{\varepsilon_2 t} + p^* \right)$$

is the optimal trajectory in the intermediate phase when the constraint is still slack. From equations (32), (33) and (34) we get the optimal values, say d_1^{**}, d_2^{**} and t^*, of the switching trajectory. Notice that different boundary values on p would yield different constrained paths based on the same optimal conditions just presented.

For now, let us look at the meaning of the conditions stated above.

Condition (32) derives, as before, from the fact that in $t = 0$ the pollution stock is p_0. The other two conditions derive from our considerations about optimal planning with inequality constraints. The time t^* is the time at which the constraint binds. Notice that the anticipative behavior of the society transforms the constrained problem into a problem of optimal stopping time.

Figure 5 Trajectories of p as a function of t



Indeed, t^* is the time when the unconstrained trajectory meets the boundary curve.

Therefore, equation (33) forces the path of p to be smooth at t^*: at the contact point where the two paths meet both trajectories must have the same slope (i.e., accumulation rate). Sometimes, this condition is called *smooth pasting* condition.

Finally, equation (34) matches the free path with the constrained path. It requires that the net variation of pollution accumulation, along the switching path, must be equal to $p_T - p_0$ allowed by the boundary values. This condition is often labelled *value matching* condition.

In short, conditions (33) and (34) assure that the trajectory of p is continuous and smooth even when the unconstrained trajectory approaches the boundary curve. This implies that future restrictions on pollution accumulation, imposed by environmental regulation, are reflected in the current decisions of society, affecting the emission policy over the entire time horizon.

Figure 5 describes a numerical simulation. It plots pollution p as a function of time t given the parameter values $\rho = 0.04$, $\delta = 0.05$, $\omega = 1$, $h = 0.5$, $\theta = 0.3$, $p_0 = 7$, $p_T = 15$, both for the constrained and the unconstrained scenarios. For this reasonable set of parameter values, p "constrained" must be larger than p "unconstrained" along the entire horizon to get $p_T = 15$

at time T. The figure shows that in the constrained regime the switching path of p follows a free trajectory before meeting the bound at time $t^* = 52$. Hereafter, the trajectory is constrained and the bounded dynamics prevails. Observe that this switching trajectory is computed employing the values d_1^{**} and d_2^{**} given by the system of equations (32), (33) and (34). Hence, the switching trajectory is different from the (always) unconstrained trajectory of p whose dynamics depends on the value d_1^* and d_2^* of the constants. Finally, note that along the two trajectories drawn in the picture the pollution growth rates are different even in the intermediate phases before the constraint binds (from $t = 0$ to $t^* = 52$). This implies that in the intermediate phases before the constraint binds the emission policy chosen by the society, to maximize the net social benefits, is different from the unconstrained policy along the entire planning horizon.

Finally, from the comparative analysis of the trajectories emerges a further key feature of the paths. The switching trajectory is continuous and smooth along the entire planning horizon. This property characterizes also the unconstrained trajectory drawn in the picture. In addition, note that for the constrained regime, as the critical time t^* draws closer, the latent constraint exerts an ever stronger influence on current emissions. In other words, as the level of pollution growth approaches the constraint, the society anticipates that future emissions will be constrained, and it will plan for this eventuality. Therefore, it is not surprising that as time tends to t^* pollution p converges smoothly to the boundary curve in such a way as to satisfy the smoothness of the trajectory along the entire pollution path.

6 Conclusion

In this paper we have shown that the presence of a latent (inequality) constraint on pollution accumulation affects the net social benefits of a society and its optimal emission policy *even when* current emissions are far below the level where the constraint binds: marginal social benefits incorporate the shadow value of future constraints, and the optimal switching trajectory between regimes is different from both the unconstrained and the constrained trajectory.

The regulatory approach has been widely used in environmental policy. It is argued that if the pollution target is properly set, and if the emitters do not violate the relevant constraints, then the target will be reached. This

argument makes regulation very attractive to environmentalists. Unfortunately, this approach has severe shortcomings. Inefficiency about regulation when agents are heterogeneous, bureaucratic and monitoring costs to obtain pollution permits or similar allowances, difficulties in designing control regimes which are income-neutral, non-neutral technology effects and credibility of environmental rules make the regulatory approach difficult to implement (Siebert, 1992; Baumol and Oates 1989).

The analysis presented in this paper adds a further element of *indeterminacy* to the problem of optimal regulation. Our admittedly crude calculation suggests that the impact of a (credible) constraint to control emissions crucially depends on the economic-ecological conditions prevailing at the time when the regulatory policy is implemented. As discussed above, in some scenarios (ecological paradise) the goals of regulation can be easily reached, although some trajectories become unfeasible depending on the time available to implement the policy. In others (ecological hell), the regulation can force the society to pursue a policy which rises emissions at the outset instead of reducing it. As a consequence, the allocative issue that has troubled Buchanan (1969) and others in the design of system to regulate externalities – such as pollution – appears to be more complex than it is believed when intertemporal elements and inequality constraints are included in the picture.

Finally, it is important to point out that the results of this paper depend on the model's assumptions. It is clear, for instance, that the optimal trajectories are affected by the functional form used to formalize the social net benefits and the form adopted for expressing the inequality constraint. Technical progress and abatement activities may be included in the model, and may change the steady state and the transition paths among equilibria. Uncertainty about environmental quality and intergenerational allocation can affect evaluation about future time and the relationship between environmental regulation and economic growth. All these issues are at the core of our future research. We would nonetheless point out that our characterization of the problem with environmental constraints remains valid in all cases.

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