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"POLLUTION CONTROL: TARGETS AND DYNAMICS"

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Pollution control: targets and dynamics

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Abstract

In this paper I study the effects of environmental regulation which establishes upper and lower binding targets to pollution emissions. Essentially, I deal with the properties of a stochastic model of pollution control in continuous-time under emission targets and uncertainty, emphasizing dynamic nonlinearities. Inside the targets pollution behaves as if it were freely floating until it hits one of the two limits. The model provides three main results. First, I show that binding targets can affect the pollution floating even when the boundaries are currently slack. Solutions of the model show that pollution becomes an S-shaped locus of the fundamentals. Second, I show that binding targets will lead to more stable pollution rate determination within the boundaries, than free floating. Finally, stabilization of pollution is related to the growth rate and volatility of fundamentals, to the sensitivity to expected changes of pollution rate and to the credibility of the authorities in defending the pollution targets.

Key words: Pollution targets; Optimal stochastic control; Uncertainty; Environmental policy.

JEL classification: L51, H23, Q28.

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1 Introduction

Over the last fifteen years the world economic system shifted from a regime of unregulated pollutant emissions, to a new and more regulated system of rules, in which authorities committed themselves to keep emissions within broad targets. The most prominent agreement was the Kyoto Protocol (1997) which set pollution targets for industrialized countries to reduce greenhouse gas emissions in the atmosphere. Later on, there were many other international agreements whose aim was the defence of the climate and environment. Following this trend, the European Commission (2009) has recently proposed binding legislation for European countries to implement the climate and energy package known as the "20-20-20" targets. These targets would cut the EU's overall emissions from the non-ETS sectors by 10% by 2020, compared with the 2005 levels.

In spite of the operative importance of these international treaties, little research has been done on how such environmental commitments would operate in practice. In particular, how do pollution emissions behave inside the targets? How would such target levels operate if they are binding? Would binding targets affect the emissions of pollution when no active policy is taking place?

In this paper I present a simple model of pollution behavior under binding targets. Essentially, I deal with the properties of a stochastic model of pollution determination in continuous-time under emission targets and uncertainty, emphasizing dynamic nonlinearities. Optimal regulation of Brownian motion is a topic, in the theory of the stochastic optimal control, which has found several applications in economics and finance (Malliaris and Brock, 1988; Svensson, 1992; Dixit and Pindyck, 1994; Turnovsky, 2000; Saltari and Travaglini 2003, 2006; Travaglini, 2008). However, over the last decade, a number of contributions reached novel insights in the field of environmental economics, using the formalities of optimal stochastic modeling. Crucial results are in Xepapades (1999), Pindyck (2000, 2002), Lin et al. (2007), Bretscher and Smulders (2007), Soretz (2007), Ansar and Spark (2009), Lin and Huang (2010, 2011), Balikcioglu et. al (2011) and Saltari and Travaglini (2011, 2012). Basically these authors derive, under economic or ecological uncertainty, conditions for optimal timing of policies whose aim is to reduce emissions of pollutants in order to maximize social welfare and/or discounted private utility and profits. In this class of models, pollution is a diffusion process which affects the objective functions in a nonlinear manner.

The outcome of this complex relationship depends strictly on the form of technology and utility functions, on the degree of market competition, on the presence of adjustment costs, on the irreversibility of inputs, and on the form and nature of the underlying stochastic process.

A previous consolidated literature on the economic effects of pollution, however, exists. Indeed, the optimal emission (allocation) of pollution is a problem that should be dated back to the work of Pigou (1920). A great deal of papers on pollution followed this original approach. But, recently some scholars have raised the question of what we mean by a *system* in which pollution-generating activities are embedded (Perman et al. 2003). This renewed approach involves bringing together economic and ecological subsystems to analyze their interactions, and to shed light on the feedbacks between pollution, environment and economic activities.

There have been ambitious attempts to formalize this relationship. Among these, one of the most appealing efforts is the so called "model of shallow lake". Using a common framework, Carpenter and Cottingham (1997), Carpenter et al. (1999), Brock and Starrett (2003) and Maler et al. (2003) have provided an explanation of why nonlinear dynamics of pollution can emerge over time. But, differently from the theory of optimal *stochastic* control, the lake model assumes that the nonlinear pattern of pollution is caused by an internal *deterministic* feedback mechanism – sometimes called internal loading – which impairs the ecosystem's ability to absorb loadings. The proponents of the lake model argue that this framework can be seen as a metaphor for many ecological problems, so that the basic framework can have a wider applicability. As said, the feedback function is deterministic and it is assumed to be S-shaped: that is for low stock of pollution (e.g., phosphorous) there is a relatively marginal damage to the water of the lake, whereas for higher stocks this contribution rises, to fall again when a maximal threshold is approached. A common functional form of internal loading is $f(P) = \frac{P^n}{1+P^n}$ with $n \geqslant 2$. The nonlinearity of this function is essential to derive a nonlinear differential equation of pollution which describes the transition among multiple steady states.

From an operative point of view, the nonlinearity has important implication for how pollution targets are set, and for the way in which pollution is mostly appropriately controlled. Some critical drawbacks weaken, however, the theoretical outcomes of the lake model. Worthy of remark is that nonlinearity of pollution is crucially determined by the nonlinear internal loading function $f(P)$. As a result, the lake model generates an *ad hoc* deterministic convex-concave dynamics of pollution which fails in explaining why actual pollution behaves the way it does, obscuring the underlying economics of the model. According to this remark, I will try to formulate a continuous-time stochastic model where the nonlinearity of emissions is the result of the optimal allocation of pollution whose dynamics depends on some "fundamentals" driving pollution, and on policies of authorities committed in defending the binding targets.

This paper is concerned with pollution targets, and with the best trajectory to respect those levels. There are two reasons why I assume the existence of two binding targets. First, in the context of uncertainty, characterizing the present framework, there may be immense difficulties in identifying economically efficient targets. Second, policy makers are likely to have multiple objectives. Efficiency matters, but it is not the only thing that matters. Therefore, targets are often chosen in practice on the basis of a mix of objectives. The mix may include technology or health considerations, regulation and welfare.

To be more specific, I study the effects of environmental regulation which establishes upper and lower binding targets to pollution emissions. Inside the targets pollution behaves as if it were freely floating until it hits one of the two bounds. The questions at the heart of the paper are: how will pollution behave inside the two targets when fundamentals are stochastic? Is the assumption of nonlinear internal loading a necessary condition to get an S-shaped pollution trajectory? Can stochastic control theory provide an optimal nonlinear allocation of pollution among boundaries?

The present model provides three main results. First, I show that binding targets can affect the pollution dynamics even when the boundaries are currently slack. Solutions of the model show that pollution emissions become an S-shaped function of the fundamentals, with the potential targets exerting a global effect on the pollution dynamics. Second, I show that binding targets will lead to more stable pollution rate determination within the boundaries, than free floating. Finally, stabilization of pollution emissions are related to the growth rate and volatility of fundamentals, to the sensitivity to expected changes of pollution rate and to the credibility of the authorities in defending the targets.

The paper is organized as follows. In section 2 we consider the effects of fundamentals and expectation on current pollution. In section 3, I derive the second order differential equation which describes the dynamics of pollution as long as pollution is strictly between the binding targets. Section 4 derives the solution at the binding targets. Section 5 outlines the conclusions of the analysis.

2 The model

In environmental economics there are different ways to model the process characterizing pollution accumulation and its effects on economic variables (Xepapadeas, 2003). Some authors argue that pollution is a by-product of production or consumption process taking place during economic activities (Brock 1973; Stokey 1996; Smulders and Gradus, 1996). In other works, it is assumed that emissions affect the flow or the accumulation of pollution in the environment (Solow, 1999; Brock and Taylor, 2004; Perman et al., 2003). Finally, pollution can have detrimental effects on utility of individuals and productivity of inputs, altering, as an externality, the features of the objective functions (Smulder and Gradus, 1996; Egli and Steger, 2007; Bretschger and Smulder, 2007; Saltari and Travaglini, 2011, 2012).

In this paper I will adopt a comprehensive approach. Pollution is assumed to be generated by a range of aggregate variables, namely, pollution generating capital (K_t^p), abatement capital (K_t^a), consumption (C), abatement technology (T) and environmental regulation (Z). I call these variables "fundamentals". In addition, I assume that the expected changes of aggregate demand ($E\Delta Y$) – where $E(.)$ is the expectation operator and ΔY is the variation of the aggregate demand Y – may influence current pollution emissions. Hence, the general formulation for the polluting generating process can be written as

$$P = p\left(K^p, K^a, C, T, Z, E\Delta Y\right) \tag{1}$$

where $P_{K^p} > 0$, $P_{K^a} < 0$, $P_C > 0$, $P_T < 0$, $P_z < 0$, $P_{E\Delta Y} > 0$ (with the subscripts denoting first order partial derivatives).

For the purposes of the model, I employ a simple linear form of equation (1). Therefore

$$P_t = \left[\alpha \frac{K_t^p - K_t^a}{T} + \beta C_t - \gamma Z\right] + \eta \left[E_t\left(Y_{t+1}\right) - Y_t\right] \tag{2}$$

In equation (2) $\alpha, \beta, \gamma, \eta$ are coefficients which measure the relative weight of inputs per unit of emissions. Pollution at time t is a by-product of capital stock K_t^p and consumption C_t. Abatement expenditures K_t^a and technology parameter T are assumed to reduce pollution; the parameter Z captures the positive effect of environmental regulation. Finally, the last addendum $E_t\left(Y_{t+1}\right) - Y_t$ provides insights about the effects of expected change of aggregate demand $E\Delta Y$ on current emissions of pollution. This latter term plays a central role in shaping the dynamics of actual pollution P_t.

To explain this, let's start from the basic case when $E_t\left(Y_{t+1}\right) = Y_t$. In this scenario aggregate demand is stable over time, and current emissions are exclusively determined by fundamentals. But, when $E\left(Y_{t+1}\right) > Y_t$, firms realize that the level of expected aggregate demand will be higher in the future $(t+1)$ than at the current time (t). This rising expectation boot firms to enlarge current production in order to satisfy the future demand. As a result, emissions of pollution anticipate the expected trend. Obviously, the opposite result occurs when $E_t\left(Y_{t+1}\right) < Y_t$. Therefore, the difference $E_t\left(Y_{t+1}\right) - Y_t$ may be interpreted as a linear, positive *or* negative, expected spill over mechanism which affect, at the current time, the ability of the aggregate system to let out pollution.

To close the model it is essential to specify the relationship between the expected growth of aggregate demand and the expected growth of pollution. Formally, I assume that

$$E_t\left(P_{t+1}\right) - P_t = \delta\left[E_t\left(Y_{t+1}\right) - Y_t\right] \tag{3}$$

where $\delta = E_t\frac{\Delta P}{\Delta Y} = \frac{E_t(P_{t+1})-P_t}{E_t(Y_{t+1})-Y_t}$ is a semi-elasticity which measures the expected marginal impact of a change of aggregate demand on pollution emissions. The accelerationist nature of this relationship is apparent: if the expected aggregate demand is above its current level Y_t, the economic system will produce pollution at a higher rate than the one realized in the previous period. Substituting by (3) in equation (2) we get the reduced form of the model

$$P_t = \left[\alpha\frac{K_t^p - K_t^a}{T} + \beta C_t - \gamma Z\right] + \frac{\eta}{\delta}\left[E_t\left(P_{t+1}\right) - P_t\right] \tag{4}$$

Equation (4) says that, given the fundamentals, an expected increase (decrease) in aggregate demand must lead to an immediate increase (decrease) in actual pollution. However, whenever there is a change in fundamentals, current pollution P_t changes immediately, affecting the expected change $E_t\left(P_{t+1}\right) - P_t$ of pollution over time. Therefore, if authorities have pollution targets they will try to manage the fundamentals in order to control the trajectory of pollution over time.

3 Pollution inside binding targets

In preparation to our shift to continuous time formulation, let's assume that the time interval is very small, of length dt, and denote the small change $E_t(P_{t+dt}) - P_t$ by $E_t(dP_t)$. Further, let's define the composite variable $x_t \equiv \alpha \frac{K_t^p - K_t^a}{T} + \beta C_t - \gamma Z$ as the *fundamentals* for the pollution. Equation (4) can be rewritten as

$$P_t = x_t + \lambda E_t\left(\frac{dP_t}{dt}\right) \tag{5}$$

where $\lambda = \frac{\eta}{\delta}$. In this formulation, the current value of pollution P_t depends on fundamentals x_t, and on its expected growth rate $E_t\left(\frac{dP_t}{dt}\right)$.

The nonstochastic problem could be approached by integrating the relationship (5) by time. If an explosive bubble path is ruled out, equation (5) should satisfy

$$P_t = \frac{1}{\lambda}\int_t^{\infty} x_s e^{-(s-t)} ds \tag{6}$$

that is the stock of pollution is given by the discounted value of expected fundamentals x_t. But, more complex is the characterization of the solution for the stochastic version of the problem.

Suppose x is the geometric Brownian motion

$$dx = \mu x dt + \sigma x dz \tag{7}$$

where μ is the instantaneous drift, σ is the instantaneous standard deviation, and dz is the increment to a Wiener process with mean of zero and standard deviation of $\sqrt{dt}$.

Absent the binding targets, equations (5) and (7) together characterize the stochastic solution. Recall that actual pollution is assumed to be limited by the commitment of the authorities to keep pollution within binding targets, whose upper limit is P_m, and whose lower limit is P_d. Whenever pollution approaches the binding targets the authorities can intervene to insure that pollution does not cross the boundaries. But, absent intervention, if the fundamentals x follow the diffusion process (7), so does the pollution. Therefore, the key to solve the model is to recognize that if fundamentals x follow a diffusion process as (7) within the bounds, all information about the future probability distribution of fundamentals is summarized in their current level x. Hence, I may write a general solution for the pollution as

$$P = p(x) \tag{8}$$

with $p(x)$ twice differentiable by assumption. Applying Ito's lemma to $p(x)$ I get

$$
\begin{aligned}
dP &= P_x dx + \frac{1}{2} P_{xx} (dx)^2 && (9) \\
dP &= P_x (\mu x dt + \sigma x dz) + \frac{1}{2} P_{xx} \sigma^2 x^2 dt && (10)
\end{aligned}
$$

since $E(dz) = (dt)^2 = 0$. Taking expectation we obtain an expression for the expected growth rate

$$\frac{EdP}{dt} = \mu P_x x + \frac{1}{2} P_{xx} \sigma^2 x^2 \tag{11}$$

Substituting by (11) in equation (5) I get the second order differential equation which describes the dynamics of pollution as long as $P = p(x)$ is strictly between P_u and P_d, that is

$$\frac{1}{2} \lambda P_{xx} \sigma^2 x^2 + \lambda \mu P_x x - P(x) + x = 0 \tag{12}$$

Since this differential equation can be thought as a function of fundamentals x, rather than time, we suppress time subscripts by design. It is easy to verify that its general solution is

$$P(x) = \frac{x}{1 - \lambda \mu} + A_1 x^{\theta_1} + A_2 x^{\theta_2} \tag{13}$$

where A_1 and A_2 are arbitrary constants to be determined, and $\theta_1 > 1$ and $\theta_2 < 0$ are the roots of the characteristic equation

$$\frac{\lambda}{2} \theta (\theta - 1) + \lambda \mu \theta - 1 = 0 \tag{14}$$

Solution (13) can be interpreted as follows. The term $\frac{x}{1-\lambda\mu}$ is the expected present value of pollution when x process is allowed to proceed without regulation, while $P(x)$ is the same when the process is regulated using the control. Therefore, the last two terms in (13) represent the additional value of the control. This general solution is not enough to describe the pollution's behavior in presence of binding targets. To do that, we have to tie down the arbitrary constants A_1 and A_2, using our assumption that the authorities limit the pollution's range to the interval $[P_d, P_m]$.

4 Optimal control of pollution

In the present model, authorities can directly intervene to defend environmental commitments. They can modify the parameters Z and T which represent, respectively, the environmental regulation and the efforts to improve abatement technology. Accordingly, authorities can also intervene imposing pigouvian taxes and subsidies to alter the decisions of firms and consumers about the optimal level of capital stocks K^p, K^a and consumption C. The compound effect of these policies is to modify the fundamentals x and the current emission of pollution P. But, these policies will also affect the expected pollution growth rate.

The key to demonstrate this is to focus on how $P(x)$ behaves approaching the lower and upper targets P_d and P_m. Let d be the minimum value of the fundamentals x when pollution reaches the lower target P_d, and m the maximum value of x when the pollution arrives at the upper target P_m. Within these two bounds pollution can float freely. However, once it has reached one of the two binding targets, its dynamics changes. This mechanism inevitably affects the floating of pollution within the bounds. As long as $P(x)$ is within the range its dynamics follows the process (5). But, when the fundamentals x reaches one of the two binding targets, the evolution of $P(x)$ becomes a modification of the process (5). This implies that as x tends to m, $P(x)$ tends to its own maximum level P_m. Similarly, when x tends to d, $P(x)$ tends to the minimum level P_d. Therefore, at the trigger values d and m the first order condition for the optimal control of $P(x)$ can be written as

$$P_x\left(d\right) = 0 = P_x\left(m\right) \tag{15}$$

This expression is often called smooth pasting condition, and usually arises as an optimality condition. It is a sufficient condition to fix the constants A_1 and A_2. Differentiating (13) with respect to x and using this condition I get

$$\theta_1 A_1 d^{\theta_1 - 1} + \theta_2 A_2 d^{\theta_2 - 1} = \theta_1 A_1 m^{\theta_1 - 1} + \theta_2 A_2 m^{\theta_2 - 1} \tag{16}$$

Hence, the explicit solution of (13) is

$$\begin{aligned} P(x) \quad = \quad & \frac{x}{1-\lambda\mu} + \left[\frac{1}{\theta_1\left(1-\lambda\mu\right)}\frac{d^{\theta_2-1}-m^{\theta_2-1}}{d^{\theta_1-1}m^{\theta_2-1}-d^{\theta_2-1}m^{\theta_1-1}}\right]x^{\theta_1} + \\ & + \left[\frac{1}{\theta_2\left(1-\lambda\mu\right)}\frac{m^{\theta_1-1}-d^{\theta_1-1}}{d^{\theta_1-1}m^{\theta_2-1}-d^{\theta_2-1}m^{\theta_1-1}}\right]x^{\theta_2} \end{aligned} \tag{17}$$

and the result is an S-shaped curve with slope

$$P_x(x) = \frac{1}{1-\lambda\mu} + A_1^*\theta_1 x^{\theta_1 - 1} + A_2^*\theta_2 x^{\theta_2 - 1} \tag{18}$$

where A_1^* and A_2^* are the solutions of the two constants in equation (17). Obviously the trigger values d and m must be chosen to attain the desired targets on P. Thus, the conditions $P(d) = P_d$ and $P(m) = P_m$ define the trigger values d and m in terms of the given targets P_d and P_m.

Note that the positive root $\theta_1 > 0$ is associated with the negative constant $A_1 < 0$, whereas, the negative root $\theta_2 < 0$, is associated with the positive constant $A_2 > 0$. The dynamics of $P(x)$ depends on three components. For small values of the fundamentals x the dominant component of $P(x)$ is the addendum with the negative root θ_2. This function is decreasing and convex. For large values of x, the component with positive root θ_1 prevails. This is negative, decreasing and concave. For intermediate values of x, the unconstrained component $\frac{x}{1-\lambda\mu}$ contributes to the increasing portion of $P(x)$. This implies an S-shaped curve of $P(x)$ like that shown in Figure 1.

To understand this geometry it is useful to rearrange (12) as

$$P(x) - x = \frac{\lambda}{2} P_{xx}\sigma^2 x^2 + \lambda\mu P_x x \tag{19}$$

This expression says that the deviation of the actual pollution $P(x)$ from the fundamentals x depends on the curvature of the fundamentals-pollution relationship, that is on the signs of the derivatives P_{xx} and P_x. Where $P(x)$ is convex the expected change of pollution is positive, and the actual pollution is above the fundamentals. Conversely, where $P(x)$ is concave the actual pollution is below the fundamental. This is an application of the Jensen's inequality.

We can further use the properties of equation (12), to explain the trajectory of $P(x)$ drawn in figure 1. $P(x)$ is an intertemporal relationship which depends on the fundamentals x and on the expected value of the pollution. Given an initial value for x, any expected change of the pollution rate implies a corresponding change in actual $P(x)$; and, correspondingly, any change in actual $P(x)$ will affect the expected pollution rate. This interdependence influences the trajectory of pollution over the range $[d, m]$. As the upper (lower) target draws closer, it exerts an ever stronger influence on actual pollution emission, and after the critical value x_c of the fundamentals, the function $P(x)$ becomes a concave (convex) function of x. As a result, *latent* binding targets will affect the pollution dynamics even when the boundaries are currently slack. Therefore, it is not surprising that as x tends to its maximum (minimum) value, $P(x)$ converges smoothly to the bounds becoming tangent at the binding targets, in such a way as to satisfy the smooth pasting condition.

It is helpful to compare the slope of $P(x)$ in expression (18) with the slope $\frac{1}{1-\lambda\mu}$ of the unconstrained trajectory drawn in figure 1. The smooth pasting condition (15) implies that $P_x(x)$ tends to zero when x goes to d and m. In addition, at the value x_c, where the two functions cross, $P_x(x_c) = \frac{1}{1-\lambda\mu} + A_1^*\theta_1 x_c^{\theta_1 - 1} + A_2^*\theta_2 x_c^{\theta_2 - 1} < \frac{1}{1-\lambda\mu}$ because A_1^* and θ_2 are negative. In other words, the slope of the function $P(x)$ is everywhere flatter than $\frac{1}{1-\lambda\mu}$, so that shocks to the fundamentals x have a dampened effect on the actual pollution in the range. Interestingly, this stabilization of emissions takes place even when the authorities are not actively defending the targets. They only have to act when the fundamentals x reach the level m or d. Therefore, what is remarkable here, is that the pollution dynamics changes its behavior not only when the targets are actually binding, but even when they are slack at the current time.

5 Conclusions

In this paper I studied the dynamics of pollution emissions in presence of environmental commitments, showing that latent binding targets may affect the emissions of pollution even when the boundaries are currently slack. In this simplified framework the expectation that authorities will act to defend the commitments exerts a stabilizing influence on pollution dynamics inside the binding range. Therefore, the S-shaped $P(x)$ locus captures the idea that environmental agreements may reduce pollution emissions and its growth rate, for any given value of the fundamentals x. Further, differently from the model of shallow lake, the nonlinear trajectory of $P(x)$ is the result of the optimal stochastic control, given the uncertain process of fundamentals x.

Basically, the stabilizing effect of pollution targets depends on the sensitivity of the current pollution to its expected growth rate, the drift and volatility of the underlying fundamentals, and the credibility of the authorities' commitment. To see this last point, let q be the probability that targets will be defended by authorities, and $1 - q$ the probability that they will not be defended. In this scenario, the expected stock of the effective pollution is given by the expression $E(P) = qP(x) + (1-q)\frac{x}{1-\lambda\mu}$. It is clear that uncertainty about the credibility of authorities will reduce the stabilizing effect of commitments. Indeed, when q is very small pollution $E(P)$ will tend to follow the fundamental x, since $qP(x) \simeq 0$. Therefore, one important implication of the model is that credibility is a crucial instrument in the hands of authorities to defend environmental commitments.

Finally, the following step of the present study is to generalize its basic outcomes. This would imply the analysis of a more complex underlying stochastic process generating fundamentals, the introduction of adjustment costs which limit the influence of the fundamentals on the actual behavior of pollution, the explicit use of corrective taxes and subsidies, and empirical analysis to test the effects of fundamentals on pollution emissions between the targets. In practice, the basic nonlinearity emphasized in the present model can be used to provide further answers on the relationship between pollution dynamics and environmental policies.

References

Ansar, J. and R. Spark (2009). "The experience curve, option value and the energy paradox", *Energy Policy*, 37, 1012-1020.

Balikcioglu M., Fackler P.L. and Pindyck R. S. (2011). "Solving optimal timing problems in environmental economics", *Resource and Energy Economics*, 33, 3, 761-768.

Bretschger, L. and S. Smulders (2007). "Introduction to sustainable resource use and economic dynamics", in L. Bretschger and S. Smulders (eds), *Sustainable Resource Use and Economic Dynamics*, Berlin, Springer, 1-16.

Brock W. A. (1973). "A polluted golden age", in Economics of Natural and Environmental Resources, edited by Smith V.L., ed, Gordon & Breach, New York.

Brock W. A. and Starrett D. (2003). "Nonconvexities in ecological management problem", *Environmental and Resource Economics*, 26, 575-602.

Brock W. A. and Taylor M. S. (2004). "The Green Solow Model", Working Paper NBER 10557, Cambridge, Massachussetts.

Carpenter, S. R., and K. L. Cottingham (1997). "Resilience and restoration of lakes" *Conservation Ecology*, 1.

Carpenter S.R., Ludwig D. and W. A. Brock (1999). "Management of eutrophication for lakes subject to potentially irrreversible change", *Ecological Applications*, 9, 3, 751-771.

Dixit, A. and R. S. Pindyck (1994). *Investment under Uncertainty.* Princeton University Press, Princeton New Jersey.

Egli, H. and T. M. Steger (2007). "A dynamic model of the environmental Kuznetz curve: Turning point and public policy", in L. Bretschger and S. Smulders (eds), *Sustainable Resource Use and Economic Dynamics*, Berlin, Springer, 17-34.

Lin, T. T., C. Ko and H. Yeh (2007). "Applying real options in investment decisions relating to environmental pollution", *Energy Policy,* 35, 2426-32.

Lin, T. T. and S. Huang (2010). "An entry and exit model on the energy-saving investment strategy with real options", *Energy Policy,* 38, 794-802.

Lin, T. T. and S. Huang (2011). "Application of the modified Tobin's q to an uncertain energy-saving project with the real options concept", *Energy Policy,* 39, 408-420.

Maler K.G., Xepapadeas A. and De Zeeuw A. (2003). "The economics of shallow lakes", *Environmental and Resource Economics,* 26, 603-624.

Malliaris A.G. and W.A. Brock (1988). *Stochastic Methods in Economics and Finance,* ed. North-Holland, Amsterdam, fourth edition.

Perman R., Ma Y., McGilvray J. and Common M. (2003). *Natural Resources and Environmental Economics,* ed. Perman Education Limited, Essex, thrird edition.

Pindyck, R. S. (2000). "Irreversibilities and the timing of environmental policy", *Resource and Energy Economics,* 22, 233-259.

Pindyck, R. S. (2002). "Optimal timing problems in environmental economics", *Journal of Economic Dynamics & Control,* 26, 1677-1697.

Pigou A. C. (1920). *The Economics of Welfare,* Macmillan, London.

Saltari, E. and G. Travaglini (2003)."How do future constraints affect current investment?", *Berkeley Journals of Macroeconomics, Topics in Macroeconomics* 3.1, 1-21.

Saltari, E. and G. Travaglini (2006). "The effects of future financing constraints on capital accumulation: some new results on the constrained investment problem", *Research in Economics* 60, 1-12.

Saltari, E. and G. Travaglini (2011)."The effects of environmental policies on the abatement investment decisions of a green firm", *Resource and Energy Economics*, 33, 3, 666-685

Saltari, E. and G. Travaglini (2011). "Optimal abatement investment and environmental policies under pollution", in *Frontiers of Economic Growth and Development,* edited by O. de La Grandville, ed. Emerald.

Smulders, S. and R. Gradus (1996). "Pollution abatement and long-term growth", *European Journal of Political Economy,* 12, 505-532.

Soretz, S. (2007). "Efficient dynamic pollution taxation in an Uncertain Environment", in L. Bretschger and S. Smulders (eds), *Sustainable Resource Use and Economic Dynamics*, Berlin, Springer, 101-125.

Solow R. (1999). "Neoclassical growth theory", in Handbook of Macroeconomics, edited by Taylor J.B. and Woodford M., vol I, Elsevier, Amsterdam.

Stokey N.L. (1996). "Are the limits to growth?", *International Economic Review*, 39, 1-31.

Svensson L.E.O. (1992). "Recent research on exchange rate target zones: An interpretation", *Journal of Economic Perspectives, 6, 119-144.*

Travaglini G. (2008). "An exact consumption rule with liquidity constraints

and stochastic income", *Economics Bulletin*, 5, 1-9.

Turnovsky S. J. (2000). *Methods of Macroeconomics Dynamics,* ed. The MIT press, Cambridge Massachusetts, second edition.

Xepapadeas, A. (1999). "Environmental policy and firm behavior: abatement investment and location decisions under uncertainty and irreversibility", *NBER Working Paper 243.*

Xepapadeas, A. (2005). "Economic growth and the environment", in *Handbook of Environmental Economics,* edited by K.G. Maler and J. Vincent, ed. North-Holland, Amsterdam.

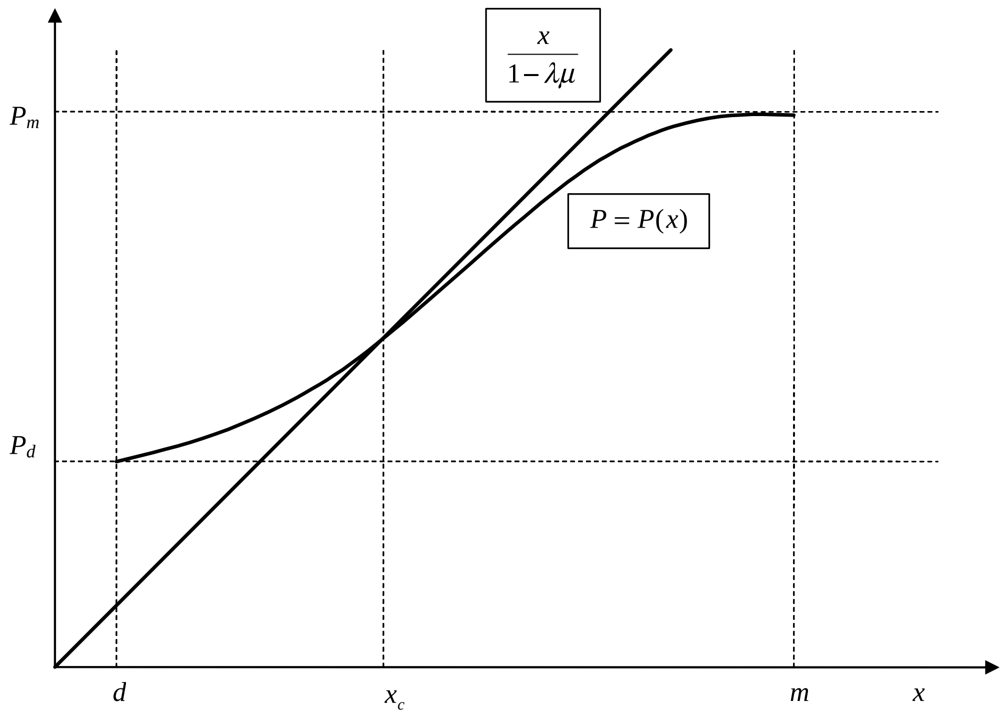

Figure 1: Pollution in binding targets.