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"Economics as a Compartmental System: A Simple Macroeconomic Example"

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Economics as a compartmental system: a simple macroeconomic example

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Abstract

The application in Economics of methods and tools originally thought for other disciplines is not new. In this paper we show how the *compartmental approach*, typical of Biology, Medicine, Chemistry, etc., can be used as a tool to represent the heterogeneity of the agents and introduce it into the dynamic models. We also show that in some cases the subdivision of the agents into compartments can be useful for the government in order to obtain some results more efficiently.

keywords: Compartmental systems; Heterogeneous agents; Fiscal policy

1 Introduction

The adoption in economics of tools taken from biology is not new. Marshall once said that "the Mecca of the economist lies in economic biology" and Georgescu-Roegen paraphrases him by saying that "biology, not mechanics, is our Mecca". The concept of *evolution*, for instance, is often applied by economists of the same idea as Marshall and Georgescu-Roegen. Sometimes biology is not directly mentioned by economists even if they use tools belonging to that field. Let us consider the huge number of applications of the Lotka-Volterra equations in explaining endogenous fluctuations of economic variables. The most famous economic application of the Lotka-Volterra equations is Goodwin [11], who applied them in a well known growth model[1].

[1] Recently the Lotka-Volterra equations have also been applied to explain dynamics in industrial clusters [8, 9].

Recently, from the criticisms of prestigious scientists (among the others the Nobel laureates Herbert Simon, Daniel Kahneman and Amos Tversky), the neoclassical hypothesis of representative agent has become less and less defensible, especially after its failure in prevent and explain the recent financial crisis. The removal of this unrealistic hypothesis opens a door to the application in economics of further concepts and instruments typical of other fields, one of them is biology. In fact, if you make the more realistic assumption that the economy is made up of people, families, firms, etc... that are in general heterogeneous, then it becomes possible to take into consideration how the different typologies of agents interact. In fact, heterogeneous agents can influence each other, can have different levels of knowledge about their environment, can use (or not use) different learning mechanisms to improve their satisfaction, and so on. The study of how one element of a group influences, and is influenced by, the elements of the other groups is quite standard in biology, so economists may turn to the toolkit of biologists to explain some behavior of groups of agents. For example, Alan Kirman [17] has shown that the behavior of humans in particular economically relevant situations is analogous to the behavior of ants that follow (i.e. imitate) the others in searching for food.

In this paper we adopt another instrument typical of other disciplines, biology in particular. We talk about the *compartmental approach*. In fact, there exists a middle way between the assumption of the unique representative agent and the assumption of completely heterogeneous individual agents. This middle way consists in aggregating agents in (meso)groups characterized by some common feature that the members of the same group share. The partial aggregation will be useful if it is reasonable to think that the characteristic feature of the groups is relevant in the economic situation we are analyzing (i.e. the groups behave differently) and the differences among the members of the same group can be deleted, making admissible the assumption of representative agents of the single group or compartment. Examples are groups of consumers of the same age or belonging to the same social class (in whatever way you define it). Looking at the supply side of the market, firms can be subdivided, for instance, by size. The latter case is the one considered in this paper. A compartmental analysis consists in specifying how the number of members of each compartment changes from period to period and from where (i.e. from which compartment) new members come and to where (i.e. towards which compartment) the exiting members are directed. This means that the modeler has at least to specify one dynamic equation for each possible (i.e. admissible) flow. Compartmental models are widely used in other fields. Let us consider for example epidemiology. Since the classic work by Kermack and McKendrick [16], there are a lot of models in which a fixed population is subdivided into compartments, for instance the compartments of Susceptible, Infected and Removed. Through compartmental analysis it is possible to characterize flows, which have a precise epidemiology meaning. The flow from Susceptible to Infected, for example, would be influenced by disease aggressiveness, while the flow from Infected to Removed would depend on the mortality of the disease, and so on. Other examples come from medicine, ecology, chemistry and pharmacokinetics.

There are not, however, so many applications in economic literature. Among these few works, we can not those of the Nobel laureate Gary Becker. Even if he does not use the word "compartment", what he does in his works is subdivide people into groups and analyze the causes of the flows and the sizes of the groups. By subdividing population into criminals and not criminals, he is able to analyze the instruments that permit the deceleration of the flow from the latter class to the former by increasing, for instance, the costs of being a criminal (probability of arrest, severity of punishment, etc...) [3]. He also gives a contribution to the study of the composition of families by subdividing people into single, married and divorced and by performing a benefits/costs analysis of being in a group or in another (or by moving from one group to another) [4, 5]. After that Von Neumann and Morgenstern [18] created a bridge between game theory and economics, the concept of *evolutionary game*, first applied by John Maynard Smith in studying evolution, began to spread among economists and is now widely used. Evolutionary games are characterized, roughly speaking, by a finite number of classes of agents and the model explores how the share of the population in each class changes according to some rules[2].

Probably, the closest approach is the one of Aoki [1, 2]. What Aoki calls *clusters* are conceptually similar to compartments. The main difference consists in the stochastic point of view taken by Aoki that leads to the introduction of quite interesting but also complicated tools like the theory of Markov processes and the Master Equation. The compartmental approach, instead, permit us to build a model described by a completely deterministic system of dynamic equation and to use more simple mathematical instruments.

The present paper moves from a work by Bischi et al. [7] in which the authors subdivide the population of firms into compartments according to their size. They proved how an exogenous shock may have different consequences on the compartments. We take the same model but use it for a different purpose. We want to show that a compartmental approach can be a good way for the government to operate in order to reach a goal as cost-efficiently as possible. In Section 2 we introduce the economic model with a representative firm. Section 3 describes how the compartmental approach can be used if we admit heterogeneity among the firms. In Section 4 fiscal policy is introduced and Section 5 concludes.

2 A New-Keynesian model with imperfect capital markets

We apply the compartmental approach to a New-Keynesian model of firm behavior. The model is the same used in [7]. Following [12] we consider a situation in which the presence of asymmetric information does not permit the firms to raise funds through the instrument of the issue of new equities. The firms have to ask loans from banks in order to finance their production completely. This

[2]See Weibull [19] for a survey.

also means the the firms need to sell at least the amount of product that allows them to pay back their debts with the banks. The alternative is bankruptcy.

Let us consider a one-to-one technology in which the output (q) is only produced by labour (l) according to the production function

$$l = q. \tag{1}$$

A second simplifying assumption is that the number of firms in the market is so high that the production of each firm is negligible. In this case the aggregate output (Q) can be considered by each firm to be equal to the production of the other firms. The single firms do not know the aggregate production and consider it a random variable. By considering a lag of one period between the production and the sale of the output and a linear demand function, we have that at each time period the price is determined by

$$p_{t+1} = a - bQ_t \tag{2}$$

with $a > 1$ and $0 < b < 1$. The aggregate production can vary between 0 and a/b in order to keep both price and output non-negative. If the value of Q is distributed according to a uniform distribution over the interval $[0, a/b]$, then the expected value of the aggregate production will be

$$E(Q) = \frac{a}{2b} \tag{3}$$

which, according to (2), corresponds to an expected price equal to[3]

$$E(p) = \frac{a}{2}. \tag{4}$$

Once the representative firm has decided the production, it uses the quantity of labour permitting to produce it. The nominal wage is fixed at W. In order to face the total cost equal to Wq, the firm must use the equity base, say A, accumulated from the previous periods. If this is not enough, it can only ask for bank loans at a fixed nominal gross return R. The difference between the cost and the equity base $Wq - A$ is called *financial gap* and corresponds to the amount of money the firm needs from banks.

Given that the revenues at period $t + 1$ depend on the price at time $t + 1$ and on the quantity produced at time t, due to the time lag the profit of the firm at time $t + 1$ will be:

$$\Pi_{t+1} = p_{t+1}q_t - R(Wq_t - A_t) \tag{5}$$

and given the expected price (4), the expected profit can be written as:

$$E(\Pi_{t+1}) = RA_t + \left(\frac{a}{2} - RW\right)q_t \tag{6}$$

[3]In (3) and (4) we have omitted the time indicators because the expected values do not depend on the period t.

Whenever the term RW is lower than the expected price, then we have a sufficient condition for the positivity of the expected profits:

$$RW < \frac{a}{2} \tag{7}$$

Following Greenwald and Stiglitz, we assume that the firm goes bankrupt at time $t+1$ if the real profit at that time (5) is negative, that is, when the following bankruptcy condition is fulfilled:

$$p_{t+1}q_t - R(Wq_t - A_t) < 0 \Rightarrow p_{t+1} < R\left(W - \frac{A_t}{q_t}\right) \equiv p^m_{t+1} \tag{8}$$

which identifies a minimum value (p^m_{t+1}) that the price may assume in order to keep the profits non-negative. In case of bankruptcy, the firms have to sustain a cost which depends on the scale of activity according to the function:

$$BC_t = cq_t^2 \tag{9}$$

with $c > 0$. It is easy to show that the bankruptcy condition on price (8) has an equivalent in terms of total output of the market:

$$Q_t > -\frac{R}{b}\left(W - \frac{A_t}{q_t}\right) \equiv Q^m_t \tag{10}$$

As Bischi et al. [7] note, conditions (8) and (10) tell us that if the competitors drive the price excessively down by producing too much, then the firms goes bankrupt. A high value of the equity base (A) and/or low costs (R and W) let the maximum value of the output rise, decreasing the probability of bankruptcy (F). In particular, if $A_t > A^M_t = Wq_t$ then bankruptcy becomes impossible because the firm is able to finance its costs completely. On the other hand, if the equity base at time t is lower than a lower threshold A^m_t, bankruptcy is certain. A^m_t is obtained by imposing the negativity of Q^m_t. In other words, any positive production of the competitors makes bankrupts our firm because the bankruptcy condition (10) is fulfilled. The threshold is:

$$A^m_t = q_t\left(W - \frac{a}{R}\right) \tag{11}$$

In general, in cases in which $A^m_t < A_t < A^M_t$ the probability of bankruptcy (F) is given by:

$$F = \frac{R}{a}\left(W - \frac{A_t}{q_t}\right) \tag{12}$$

2.1 The output choice

Regarding the production decisions of the firm, we must distinguish between two cases. When the firm has enough equity base to avoid bank loans and hence to delete the risk of bankruptcy completely (i.e. when $A_t > A_t^M$), then it produces a quantity given by:

$$q_t = \frac{A_t}{W} \tag{13}$$

Whenever $F > 0$ then the possibility of bankruptcy must enter into the expected profits equation through the extra-costs the firm would incur in such a case:

$$E(\Pi_{t+1}) = RA_t + \left(\frac{a}{2} - RW\right)q_t - Fcq_t^2 \tag{14}$$

The quantity that maximizes (14) is the following:

$$q_t = min(\bar{q}, q^M) \quad where \quad q^M = \frac{a}{4c}\left(\frac{a}{RW} - 2\right) + \frac{A_t}{2W} \tag{15}$$

In (15) a full capacity ceiling equal to $\bar{q}$ is assumed. Putting together (13) and (15) we have that the production of the firm at time $t+1$ is given by:

$$q_{t+1} = H(A_t) = \begin{cases} \frac{A_t}{W} & if \quad A_t \geq A_t^M \\ min(\bar{q}_t, q^M) & if \quad A_t < A_t^M \end{cases} \tag{16}$$

We are left only to specify how the equity base evolves. At each time period the equity base is equal to the current profit, that is:

$$A_{t+1} = p_{t+1}q_t - R(Wq_t - A_t) \tag{17}$$

3 The compartmental approach

In this section we apply the compartmental approach to the model presented in the previous section. The implementation follows Bischi et al. (1998). Given the high number of possible applications of the compartmental techniques, there are a lot of papers explaining how to introduce compartments in a model. We point the interested reader to [6, 10, 13, 14, 15] for a deep analysis of compartmental systems. For our purposes we can subdivide the implementation of compartments into three phases:

- **definition and number of compartments**;
- **definition of admissible flows**;
- **dynamics**.

3.1 Definition and number of compartments

Jacquez and Simon [14] define a compartment as "... an amount of some material that is kinetically homogeneous". In our economic model the term *kinetically homogeneous* can be read as identifying groups of firms which behave in a similar way. Firms can be grouped together in many ways. You can consider their age, their location, their size, etc... In making production decisions, a natural way of grouping firms is by size. Here there is another problem: how can you measure the size of a firm? In the economic literature there are a lot of measures used to define the size of a firm. The number of workers, the amount of sales, the share capital are all possible measures. Common sense suggests that a big firm should produce and sell a high amount of goods but above all a firm is big if it achieves high profits. In our model, this condition means that the bigger the firm, the higher its equity base (A). We decided to use the equity base as a measure for the size of the firms.

Now we have to decide the number of compartments grouping the whole population of firms. As an initial approximation we can divide firms into *small* and *large*, creating two compartments. By limiting the number of compartments to two we exclude the entry of new firms as well as the exit of firms from the market due to bankruptcy, which is a fundamental part of our model. We can solve this problem by adding a third compartment representing the firms that are outside the market. Let us call compartments 1,2,0 the small, large and outside of the market firms, respectively. Now we have enough compartments but, in order to proceed, we need to define their borders. In particular, given that we have chosen the equity base as the key-feature of the firms belonging to the same compartment, we must decide which range of equity base is appropriate to the compartments. In general we can consider both endogenous and exogenous borders. To keep the analysis simple we consider exogenous borders. So, a firm is considered small if its equity base belongs to the interval (A_0, A_1). The firm is large if it has a value of A higher than A_1, while it goes out of the market if its equity base decreases below A_0. From now on we indicate with $A_{s,t}$ and $A_{l,t}$ the equity base at time t of the representative small firm and large firm, respectively. Similarly, $x_{s,t}$ and $x_{l,t}$ denote the number of the members at time t of the small firm compartment and the large firm compartment, respectively.

3.2 Flows between compartments

The fundamental characteristic of a compartmental model is that it is dynamic. Its dynamics consists of the flows between compartments that make the number of elements (in our case firms) inside each compartment a dynamic variable. The admissible flows depend upon the assumptions and the interpretation of each movement. For instance, in the case in which the firms were divided by age into young and old, the flow from the out-of-the-market compartment and the compartment of the old firms should not be admissible. In fact, a firm leaving the non-existing state must be located only in the young firm compartment because of the economic interpretation given to the compartment and to the

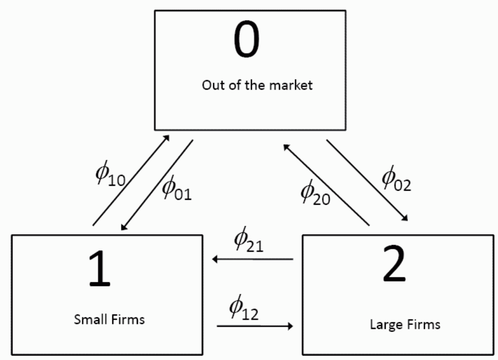

FIGURE 1

flows. In our case, all possible flows are admissible. Denoting by ϕ_{ij} the flow of firms from compartment i to compartment j, these are the interpretations of each flow:

- ϕ_{01} represents small size firms entering the market;
- ϕ_{02} represents large size firms entering the market;
- ϕ_{12} represents small firms that grow to the point of becoming large;
- ϕ_{21} represents large firms that shrink to the point of becoming small;
- ϕ_{10} represents the flow of small firms that go bankrupt;
- ϕ_{20} represents the flow of large firms that go bankrupt.

In Fig. 1 we have a graphic representation of the compartments and the flows.

3.3 Dynamics

In this subsection we specify the equations regulating the admissible flows. We have to define when each flow is activated and its intensity. We deal with three compartments, two of which have a representative agent. We have decided to adopt the equity base as critical parameter and we have also decided to fix exogenously the thresholds separating the three compartments. Given these assumptions the flows at time $t + 1$ can be activated as follows:

- ϕ_{01} is active if the equity base of the representative small firm at time t is higher than its previous period value (i.e. if $A_{s,t} - A_{s,t-1} > 0$). In other words, incoming firms realize that small firms are increasing their net worth and decide to start with a low investment;

- ϕ_{02} is active if the equity base of the representative large firm at time t is higher than its previous period value (i.e. if $A_{l,t} - A_{l,t-1} > 0$). New firms think that higher investment can be slowly paid back thanks to the increasing profits for large firms in that market;
- ϕ_{12} is an active flow whenever the representative small firm reaches in t an equity base higher than the threshold A_1;
- ϕ_{21} is an active flow whenever the equity base of the representative large firm at time t goes below the lower threshold A_1;
- ϕ_{10} is active when small firms go bankrupt. This happens when their equity base is lower than A_0;
- ϕ_{20} is active when large firms go bankrupt. This happens when their equity base is lower than A_0.

In order to regulate the intensity of the flows we have to remember that the firms still are heterogeneous inside each compartment. This means, for instance, that even if the representative small firm goes bankrupt, this does not necessarily imply that all the small firms go out of the market. A simple assumption consists in considering the intensity of the flows proportional to the signal activating it. Continuing with the example of representative small firm going bankrupt, we assume that the flow ϕ_{10} is given by $a_{10}(A_0 - A_{s,t})x_{s,t}$, where a_{10} is a positive intensity parameter. Similar mechanisms work for the other flows. Summing up, we have two dynamics equations for the equity base of the two representative firms, two dynamic equations for the number of the members of each compartment and the dynamic equation of the market price. The system is described by the following five-dimensional discrete-time dynamical system:

$$\begin{aligned}
x_{s,t+1} &= x_{s,t} + a_{01}\max(A_{s,t} - A_{s,t-1};0) - a_{01}\max(A_0 - A_{s,t};0)x_{s,t} \\
&\quad - a_{12}\max(A_{s,t} - A_1;0)x_{s,t} + a_{21}\max(A_1 - A_{l,t};0)x_{l,t} \\
x_{l,t+1} &= x_{l,t} + a_{02}\max(A_{l,t} - A_{l,t-1};0) - a_{02}\max(A_0 - A_{l,t};0)x_{l,t} \\
&\quad + a_{12}\max(A_{s,t} - A_1;0)x_{s,t} - a_{21}\max(A_1 - A_{l,t};0)x_{l,t} \\
A_{s,t+1} &= p_{t+1}H_s(A_{s,t}) - R[WH_s(A_{s,t}) - A_{s,t}] \\
A_{l,t+1} &= p_{t+1}H_l(A_{l,t}) - R[WH_l(A_{l,t}) - A_{l,t}] \\
p_{t+1} &= a - b\{\max[H_s(A_{s,t})x_{s,t} + H_l(A_{l,t})x_{l,t};0]\}
\end{aligned} \tag{18}$$

It is not easy to study analytically the system. We prefer to limit ourselves to analyse some numerical experiments in the next section.

4 Fiscal policy with compartments

The system represented by (18) lends itself to different purposes. Bischi et al. [7], for instance, test the response of the system to a nominal shock (an exoge-

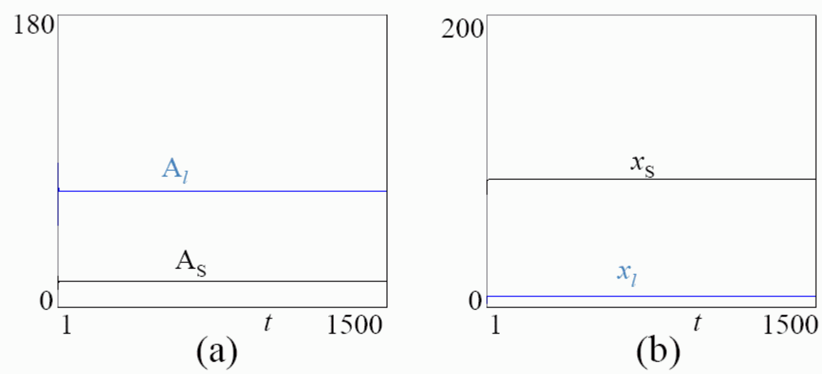

FIGURE 2

nous change of the price level). They show how the shocks may be persistent and break a situation of symmetry[4].

We want to show how the subdivision into compartments of the firms can be used by the fiscal policy authority to improve an undesirable situation. The following example is obtained by fixing the demand parameters $a = 20$ and $b = 0.05$. The cost parameters are $c = 1$ and $R = 0.01$. The bankruptcy threshold is given by $A_0 = 0.5$ while the two compartments are separated by a value of the equity base $A_1 = 40$. That is, a firm is large if its equity base is greater than 40 and goes bankrupt in the case its equity base goes below 0.5. The parameters a_{01} and a_{02} are assumed equal to 0.7 and 0.1, respectively. This last assumption implies a higher propensity of new born firms to be small rather than large. The parameters a_{10}, a_{20}, a_{12} and a_{21} are all fixed to 0.2. Fig. 2 represents what happens under this parameter configuration if the nominal wage W is equal to 9.2. As we can see the system rapidly settles. From an initial number of firms equal to 40 for both the compartments, large firms drop to around 8 whereas the compartment of small firms increases to about 90 firms. The equity base of the representative small firm is close to 16, while the large firm is characterized by an equity base higher than 70. Let us consider now the consequences of a nominal shock. At period 120 we introduce an exogenous additive shock to the price level. Fig. 3 shows how the shock is absorbed in a few periods-if we look at prices. Fig. 4 reveals that the real effects of the

[4]In their experiment they moved from a situation in which the compartments are characterized by the same number of firms. Large firms become much more numerous than small firms as a consequence of the shock.

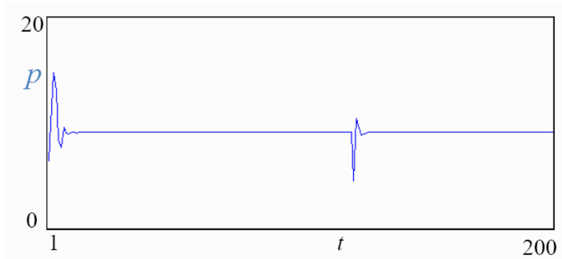

FIGURE 3

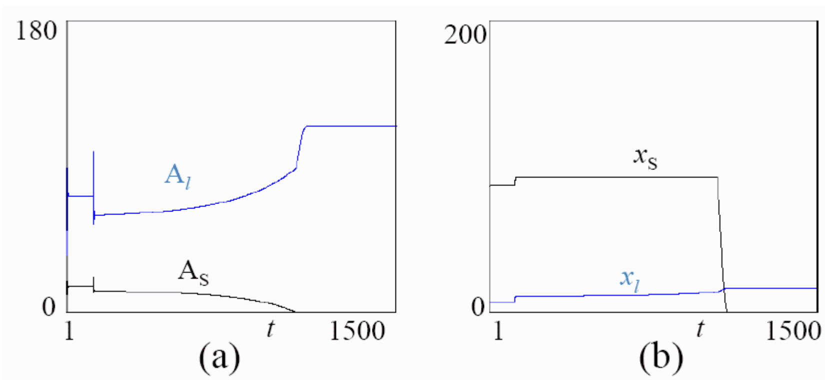

FIGURE 4

nominal shock are huge[5]. The equity base of the small firms starts to decrease immediately after the shock until it becomes smaller than the lower threshold A_0 (Fig. 4(a)), then small firms go gradually bankrupt leaving the market only to the large firms(Fig. 4(b)). Let us consider the case in which the market situation before the nominal shock is more desirable than the oligopoly that appears after it[6]. If the few large firms collude than higher profits of the firms will substitute consumers' surplus and this should be an undesirable consequence. Is it possible to avoid the situation represented by Fig. 4? Let us consider the nominal wage as made up by more than one component. It is reasonable to think that one component is labour taxation. When, after the shock, the fiscal policy authority realizes that the equity base of the small firms is dangerously decreasing it can

[5]This is one of the main results obtained by Bischi et al. [7].

[6]We could endogenize this effect by let the firms realize that they have power on the market price when they are just a few, permitting them to collude .

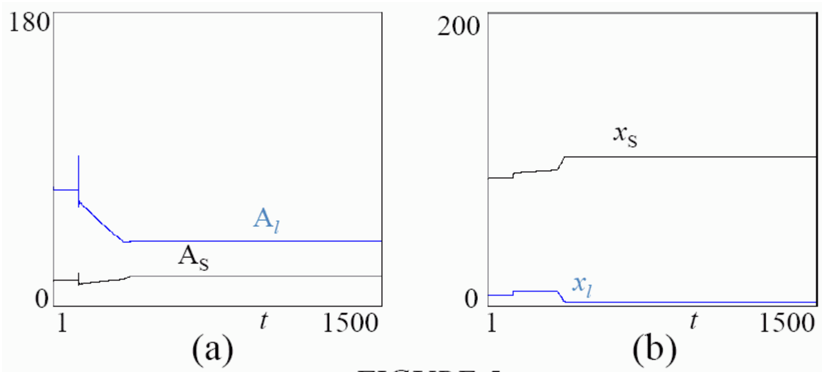

FIGURE 5

decide to cut labour taxes temporarily in order to help the firms. The authority can reduce taxes for all the firms, but it can also decide to apply the fiscal discount only to small firms. In the latter case, the authority itself is aware that the firms can be subdivided into compartments according to their size and that only the smaller firms risk going bankrupt. Fig. 5 shows what happens by applying a fiscal discount to the small firms such that from period 130 to 350 the nominal wage they have to pay is reduced to 9.17 instead of 9.2. As we can see, the reduction of labour taxes is enough to help the small firms stabilize their equity base at a level higher than the bankruptcy threshold (in the numerical simulation of Fig. 5 $A_s \simeq 18$) stopping its reduction. At the same time the measure adopted by the fiscal policy has the indirect effect of reducing the equity base of the large firms. In other words, the variance of size between small and large firms has been reduced by the fiscal policy authority. The number of small firms is increased whereas large firms are less than before the shock (Fig. 5(b)). On the whole, the total number of firms in the market is higher. A similar result would have been reached had the fiscal policy authority applied the same discount to all the firms. The main difference concerns the fiscal revenues during the period in which the discount is effective. By reducing the taxes only to one compartment (the small firms) the government obviously accepts a lower amount of fiscal revenues with respect to a general discount. We can say that the *compartmental measure of fiscal policy* is more efficient than the measure adopted by not taking into account the subdivision of firms in compartments[7].

[7]Note that the market situation after the application of the fiscal discount is very similar in the two cases, so the relevant difference in the fiscal revenues are those concerning the periods in which the measures are applied.

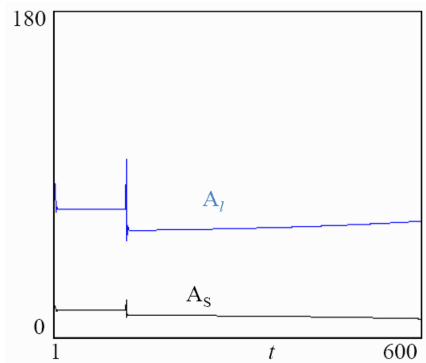

FIGURE 6

4.1 Timing of the intervention

If we look at the effects of the nominal shock in Fig. 4, we note that the exit of the small firms from the market is not gradual. It happens quite suddenly, in a few periods around $t = 750$. This fact is very important because it makes it difficult for the government to forecast and prevent undesirable effects of the shock. This concept is cleaver if we look a Fig. 6, which represents only the first 480 periods that follow the price shock. Looking at the absolute value of the equity base of the two groups of firms, it may seem that the consequences of the shock are not very significant. This is the kind of reasoning that a wise government has to avoid. In fact, the main difference between the time series of the equity bases before and after the shock is the decreasing trend of the small firms' equity base. Before the shock the equity base is stable but after the shock it starts to decrease. This is the warning that the government must heed in order to adopt a well-timed and efficient fiscal policy. In cases in which the government realizes too late that something undesirable is going to happen, then it becomes quite expensive, if not impossible, to prevent it.

5 Conclusions

In this paper we have shown how the compartmental approach typical of Biology and other disciplines can be applied to an economic model. We have proposed a simple example based on a model of firm's behavior inspired by a paper by Greenwald and Stiglitz [12]. This approach actually represents a deterministic alternative to the aggregation methods developed by Aoki [1, 2]. The main result of this exercise consists in showing how the heterogeneous agents hypothesis, whenever it is taken into consideration by the authorities, may be a useful instrument in order to adopt more efficient policy measures. In the future we intend to apply this tool to more sophisticated models. Nevertheless the implementation of the compartmental approach in a model that is as simple as possible, can be useful to understand better the mechanisms through which some phenomena emerge only at an aggregate level in agent-based modeling. In fact, the compartmental approach permits the modeler to subdivide the distance between the representative agent hypothesis and the completely heterogeneous agents hypothesis into a wide spectrum of models which may be closer to one of the extremes or the other according to the number of compartments considered.

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