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"Consumer Cooperatives and Welfare in a Mixed Oligopoly"

- Marco Marini, (U. Urbino & CREI, U. Roma III)
- Alberto Zevi, (U. di Roma La Sapienza)

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CONSUMERS COOPERATIVES AND WELFARE IN A MIXED OLIGOPOLY

MARCO MARINI AND ALBERTO ZEVI

Abstract. Consumer co-operatives constitute a highly successful example of democratic forms of enterprises operating in developed countries. They are usually organized as medium or large-scale firms competing with profit-seeking firms in retail industries. In this paper we model such a situation as a mixed oligopoly in which consumer co-operatives maximize consumer-members' utilities and distribute them a patronage rebate on their goods purchase. We show that when consumers possess quasilinear preferences over a bundle of symmetrically differentiated goods and firms operate with a linear technology, the presence of consumer co-operatives positively affects all industries output and social welfare. The effect of Co-ops on welfare is shown to be more significant when goods are either complements or highly differentiated and when competition is *à la* Cournot rather than *à la* Bertrand.

Keywords: *Consumer Co-operatives*, *Profit-maximizing Firms*, *Mixed Oligopoly*.

JEL Classification Number: L21, L22, L31

1. Introduction

Since 1844, the Rochdale pioneers' idea of cooperation has spread around the world and today more than 700 millions cooperators are active in 100 countries (ICA, 2006). Among the various cooperative forms of enterprises, consumer cooperatives (henceforth Coops) are typically firms which operate in retail industries pursuing the institutional objective to act on behalf of their consumer-members.[1] Today these organizations represent one of the most successful realities among existing democratic and participative forms

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Corresponding author: Marco Marini, Department of Economics, Università di Urbino "Carlo Bo" and CREI, Università Roma III. Address: via Saffi, 42, 60129, Urbino. Tel. +39-0722-305557; Fax:; +39-0722-305550 E- mail: marco.marini@uniurb.it.

Alberto Zevi, Università di Roma La Sapienza.

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[1] Usually consumer-members are entitled to elect their representatives who participate to the assembly electing the (professional or non professional) managers running the firm.

of enterprise, able to compete against large private companies. Formed through a discontinuous process of sequential waves (see, for instance, for a brief account of the US case, Finch, Trombley & Rabas, 1998) in many countries Coops are well established without in general possessing a dominant position in retail industries, with a few exception as, for instance, Switzerland, Finland and Japan. One of the most world's well known consumer cooperative is the *Cooperative Group* in UK, which provides a variety of retail and financial services. Japan is also known to possess a very relevant consumer Cooperative movement with over 23,5 millions members and with retail cooperatives alone scoring in 2006 a turnover of about 374 billions of yen (Japanese Consumers' Cooperative Union, 2006). In Italy, the largest group of consumer cooperatives represents today a serious competitor to private companies operating in retail industry. Among the top 30 Italian retail companies, 9 are consumer cooperatives, with a recorded turnover of about 12,9 billions of euro in 2009, corresponding to around 18% of the total market share (E-coop 2010). The European Association of Consumer Cooperatives estimates that in Europe approximately 3,200 consumer cooperatives are active, with a total turnover of 70 billions of euro, employing 300,000 workers and serving about 25 millions of consumer-members (Euro Coop, 2008).

So far, the economic literature on consumer cooperatives (e.g. Enke 1945, Anderson, Porter & Maurice 1979, 1980, Ireland and Law 1980, Sexton 1983, Sexton and Sexton 1987, Farrell 1985, and more recently, Hart and Moore, 1996, 1998 and Mikami, 2003, 2010) has mainly focussed on the behaviour of these firms under either perfect competition, monopoly or monopolistic competition. However, in modern economies Coops compete strategically with traditional profit-maximizing firms (henceforth PMFs), therefore giving rise to a specific instance of *mixed oligopoly.*[2]

To the best of our knowledge, there are no existing contributions dealing specifically with mixed oligopoly between Coops and PMFs. An exception is Goering (2008) who presents a homogeneous good duopoly model between a PMF and a non profit firm assumed to maximize a parametrized combination of firm's profit and consumers' surplus. Moreover, there is a wide related literature dealing with mixed duopoly with a labour-managed firm *à la* Ward (1958) and Vanek (1970) competing with a PMF (see, for instance Law and Stewart, 1983 and Cremer and Cremér, 1992) as well as a wide literature on agricultural cooperatives under imperfectly competitive markets (Rodhes, 1983, Fulton, 1989, Sexton, 1990) or under mixed duopoly with either homogeneous (Tennbakk, 1992) or vertically differentiated good (Fulton and Giannakas, 2001). However, in general labour-managed and

In large Co-ops the assembly elects a board of directors which, on their behalf, controls the management.

[2]See, for instance, Delbono and De Fraja (1990) for a survey of mixed oligopoly models with one or more publicly-owned firm competiting with PMFs.

farmers' cooperatives are thought to behave differently from an organization owned by its consumers.

The major purpose of our paper is to present a taxonomy of the results obtained in a mixed oligopoly market in which an arbitrary number of PMFs and Coops compete strategically either in quantities or in prices and goods are differentiated. We model a Coop as a firm which maximizes the utility of consumer-members and distributes them all its net surplus as a patronage rebate on their purchased goods. As a result, a Coop is shown to set in equilibrium a price equal to its average cost, thus affecting the equilibrium behaviour of rival PMFs. All firms are assumed to possess a constant returns of scale technology and therefore in equilibrium every Coop sets a price equal to its constant marginal cost. In our model, the marginal cost pricing rule emerges endogenously. This pricing rule makes the results of our model comparable to those obtained in mixed oligopoly models with state-owned and PMFs (for instance, Cremèr, Marchand and Thisse,1998 and De Fraja and Delbono, 1998). Moreover, the constant average cost assumption has the advantage to overcome many of the issues on the stability of Co-ops' membership.[3]

Our model reaches a few relevant relevant results. We show that, under consumers' quasilinear preferences and firms' linear technology, the presence of Coops in the market affects positively the total industry output as well as the total industry welfare (and negatively the market prices). Under Cournot oligopoly with homogeneous goods it is shown that the presence of Coops pushes all PMFs out of the market (or, alternatively, oblige them to behave as perfectly competitive firms) and by this way maximizes the total market welfare. When goods are differentiated, the effect of Coops on welfare is shown to be more significant when goods are either complements or highly differentiated and when competition is *à la* Cournot (in quantities) rather than à la Bertrand (in prices). According to these results, we should expect to see consumer cooperatives more often in markets with such features.

The paper is organized as follows. Section 2 introduces the model. Section 3 and 4 present the main results of mixed oligopoly with quantity and price competition. Section 5 concludes.

2. The model

2.1. **Consumers Preferences.** The demand side of the market is represented by a *continuum* of identical consumers ($i = 1, .., I$) of unitary total measure possessing quasi-linear preferences on n symmetrically differentiated goods[4] x_k, ($k = 1, ..., n$) and an outside good y. These preferences

[3]See on this matter, Anderson, Maurice & Porter (1979), Sandler & Tschirhart (1981), Sexton (1983) and Sexton & Sexton (1990).

[4]Each good can also be interpreted as a bundle of goods sold by every firm in the market.

can be expressed, for each consumer, by a utility function $U^i : \mathbb{R}_+^{n+1} \to \mathbb{R}_+$ defined over the n products and a separate *numeraire* good y, as

$$U^i(x_1, x_2, ., x_k, .., x_n, y) = u^i(x_1, x_2, ., x_k, .., x_n) + y^i \tag{2.1}$$

in which $u^i(.)$ is smooth, increasing and strictly concave in every good x_k.[5]

If the available income of every consumer (denoted $\overline{y}^i$) is high enough, the downward-sloping individual inverse demands can be obtained from the first-order conditions for the maximization of problem (2.1) under budget constraint, as

$$p_k = \frac{\partial U^i(x_1, ., x_k, .., x_n, y)}{\partial x_k}, \text{ for } x_k > 0 \text{ and } k = 1, 2, ...n.$$

2.2. **Industry.** In the retail industry we assume that in general n firms supply n differentiated goods (or bundles of goods) whose m are supplied by consumer cooperatives and $(n - m)$ by traditional profit-maximizing firms. We will denote by $M \subset N$ the subset of firms which are Coops. As usual, PMFs firms are assumed to maximize profits as

$$\pi_k(x_1, ...x_n) = p_k(x_1, x_2, ...x_n)\, x_k - c_k(x_k). \tag{2.2}$$

Let every firm' total costs be linear and, for simplicity, firms do not bear fixed costs. As anticipated, Coops are assumed to act on behalf of their consumer-members, therefore maximizing the utility function of every i-th representative consumer-member. More specifically, every member of the Coop receives a patronage refund in proportion to the goods he has purchased over the firm's total sales. The following objective-function is therefore assumed for every Coop $j \in M$,

$$\left\{\begin{array}{ll} \max\limits_{x_j^i} U^i\left(x_1^i, x_2^i, .., x_n^i, y^i\right) & \text{s.t} \\ \sum\limits_{k=1}^{n} p_k(x_1, ., x_n)\, x_k^i + y^i \leq \overline{y}^i + \sum\limits_{j \in M} \dfrac{x_j^i}{x_j}\left(p_j(x_1, ., x_n)\, x_j - c_j(x_j)\right). & \end{array}\right. \tag{2.3}$$

Analogously, when price instead of quantity is the choice variables of every firm, the above objective functions for PMFs and Coops can be expressed as a function of the price vector $(p_1, p_2, ..p_n)$.

For every $j \in M$ the FOC of problem (2.3) provides the following condition for an interior maximum,

$$\frac{\partial U^i\left(x_1^i, .., x_n^i, y^i\right)}{\partial x_j} = \frac{c_j(x_j)}{x_j}, \tag{2.4}$$

as long as the equilibrium price is sufficiently high to generate non negative profits, that is, for $p_j(x_1^*, .x_n^*) \geq \frac{c_j(x_j^*)}{x_j}$. The meaning of (2.4) is that a Coop, acting on behalf of its consumer-members, sets its quantity to equate every consumer-member's willingness to pay for good j to the average cost

[5]The Hessian of U^i is negative definite.

of this good,[6] in order to distribute the maximum consumers' surplus to its members. Since here firms possess a constant returns of scale technology, every firm's marginal cost will be just equal to its average cost. The use of a differentiated goods oligopoly is a feature which distinguishes our setup from the numerous existing mixed oligopoly models in which, either state-owned firms (Marchand *et al.* 1998 for instance) or no-profit organizations (Fulton and Giannakas, 2001, Goering, 2008) compete with PMFs, but goods are in general homogeneous.

It can be useful to compare problem (2.3) to the case in which a social manager is assumed to coordinate all existing consumer cooperatives in order to maximize the utility of all existing consumers. In this case we should expect that FOC becomes, for every $j \in M$

$$\frac{\partial U^i\left(x_1^i, .., x_n^i, y^i\right)}{\partial x_j} = \frac{c_j(x_j)}{x_j} + \sum_{r \neq j} \frac{\partial p_r\left(x_1, .., x_n, y\right)}{\partial x_j}. \tag{2.5}$$

Therefore, for the collectivity of consumer-members the best pricing policy would be to set a price lower than average cost when product are substitute ($\frac{\partial p_r(x_1,..,x_n,y)}{\partial x_j} \leq 0$) and higher than average cost when products are complements ($\frac{\partial p_r(x_1,..,x_n,y)}{\partial x_j} \geq 0$).

3. Oligopoly with Quantity Competition

In order to study the implications of the simultaneous presence of both PMFs and Coops in an oligopolistic market, let the following utility function represent the preferences of every i-th consumer in the economy, $(i = 1, .., I)$:[7]

(3.1)
$$U^i\left(x_1, x_2, .., x_n, y\right) = \alpha \sum_{k=1}^{n} x_k^i - (1/2)\left[\sum_{k=1}^{n}\left(x_k^i\right)^2 + \beta \sum_{k=1}^{n} x_k^i \sum_{r \neq k} x_h\right] + y^i$$

where $\alpha > 0$ and $\beta \in [1/(1-n), 1]$ represents the degree of product differentiation. For $\beta = 0$, goods are independent and for $\beta = 1$ goods are perfect substitute. Moreover, for $\beta < 0$ goods become complement. The FOCs of (3.1) yield linear inverse demand functions for every good $k = 1, 2, .., n$ given by

$$\alpha - x_k - \beta \sum_{r \neq k} x_r = p_k \quad \text{for } x_k > 0. \tag{3.2}$$

It is easy to see that the FOC of problem (2.3) yield the following conditions for every Coop producing the j-th good

[6] Similarly, if competition is in prices, a Co-op will set a price equal to the average cost.

[7] See Shubik ans Levitan (1971), Vives (1984) and Dixit (1983) for further details on this utility specification.

$$\alpha - x_j - \beta \sum_{r \neq j} x_r = \frac{c_j(x_j)}{x_j} \text{ for } x_j > 0. \tag{3.3}$$

Expression (3.3) can be considered as the FOC of every Coop acting on behalf of consumer-members.

3.1. The Benchmark Case: Oligopoly with all PMFs.

As a benchmark case of our results we start illustrating the case in which all firms are PMFs and choice variables are quantities. Let all firms $k = 1, 2, ...n$ possessing identical strategy sets $X_k = [0, \infty)$ and identical technology, expressed by a linear cost function, $c_k(x_k) = cx_k$ with $0 < c < \alpha$. When firms are all PMFs they simply maximize their profit with respect to the quantity of the k-th good,

$$\pi_k(x_1, x_2, ..., x_n) = (\alpha - x_k - \beta \sum_{r \neq k} x_r)x_k - cx_k. \tag{3.4}$$

Solving the maximization problem, the following best-replies are obtained for every k-th PMF,

$$x_k(x_{-k}) = \frac{1}{2}(\alpha - \beta x_{-k} - c)$$

where $x_{-k} = (x_1, x_2, .., x_{k-1}, x_{k+1}, .., x_n)$ and therefore pure-PMF Nash equilibrium quantities $(\overline{x}_1, \overline{x}_2, ..., \overline{x}_n)$ are easily obtained as

$$\overline{x}_k = \frac{(\alpha - c)}{2 + \beta(n-1)} \tag{3.5}$$

for each k-th PMF and prices are

$$p_k(\overline{x}_1, ., \overline{x}_n) = \frac{\alpha + c + \beta c(n-1)}{\beta(n-1) + 2}.$$

It easy to see that, for $\beta = 1$, (3.5) becomes the usual Cournot solution with homogenous good ($\overline{x}_k = (\alpha - c)/(n+1)$), while for $\beta = 0$ the goods are independent and firms act monopolistically ($\overline{x}_k = (\alpha - c)/2$).

3.2. Mixed Cournot Oligopoly.

Let us imagine now that m firms in the market ($m \leq n$) transform into Coops accepting all consumers as their members. The market turns therefore into a mixed oligopoly in which m Coops compete against $(n - m)$ standard PMFs. Let $M \subset N$ denotes the set of all j-th Coops and then $N \backslash M$ is the set of all remaining h-th PMFs.

Using (3.3) and (3.4), the following best-replies are obtained for Coops and PMFs respectively, as a function of the quantities of the other type of firms only:

$$x_j\left(\sum_{h \in N \backslash M} x_h\right) = \frac{\alpha - \beta(n-m)x_h - c}{1 + \beta(m-1)}, \forall j \in M, \tag{3.6}$$

$$x_h\left(\sum_{j\in M} x_j\right) = \frac{\alpha-\beta m x_j - c}{2+\beta\left(n-m-1\right)},\ \forall h\in N\backslash M. \tag{3.7}$$

The mixed oligopoly Nash equilibrium output, denoted x^* with m Coops and $(n-m)$ PMFs is therefore obtained as

$$x_j^* = \frac{\left(2-\beta\right)\left(\alpha-c\right)}{2+\left(n+m-3\right)\beta-\left(n-1\right)\beta^2},\ \forall j\in M, \tag{3.8}$$

and

$$x_h^* = \frac{\left(1-\beta\right)\left(\alpha-c\right)}{2+\left(n+m-3\right)\beta-\left(n-1\right)\beta^2},\ \forall h\in N\backslash M, \tag{3.9}$$

with corresponding equilibrium price

$$p_h\left(x_1^*,.,x_n^*\right) =$$

$$= \frac{\alpha+c+\beta\left(c\left(2m+n-2\right)-\alpha\left(m+1\right)\right)+\beta^2\left(m\alpha-c\left(n+m-1\right)\right)}{2+\left(n+m-3\right)\beta-(n-1)\beta^2}$$

for every h-th PMF and

$$p_j\left(x_1^*,x_2^*,..,x_n^*\right) = c.$$

for every j-th Coop.

It is easy to see that, in general, if goods are perfect substitute ($\beta=1$) the model yields the extreme prediction that the presence of even just one Coop in the market pushes PMFs out of the market. This could, alternatively, be interpreted as if the presence of Coops has obliged all PMFs to adopt a perfectly competitive behaviour, in order to stay in the market. In either way, since market equilibrium price coincides with all firms' average and marginal cost, every consumer's willingness-to-pay for the homogeneous good is just equal to every firm's marginal cost of production and this implies a welfare maximization (since $u'=c$). These results are condensed in the next proposition. Finally, note that the total market output at the mixed oligopoly $X^*=\sum_{k=1,..,n} x_k^*$ is equal to

$$X^* = m x_j^* + (n-m)x_h^* = \frac{\left(\alpha-c\right)\left(n(1-\beta)+m\right)}{2+\left(n+m-3\right)\beta-\left(n-1\right)\beta^2} \tag{3.10}$$

For $m=0$ the above expression coincides with the pure n-PMFs pure oligopoly $X^*(m=0)=\frac{n(\alpha-c)}{2+\beta(n-1)}$ and for $m=n$ the expression turns into the pure Coop total quantity, with $X^*(m=n)=\frac{n(\alpha-c)}{1+\beta(n-1)}$. It is easy to see that a pure Coop oligopoly yields a higher output than a pure PMF oligopoly. Moreover, expression (3.10) shows that the output grows monotonically with the number of Coops in the market.

Proposition 1. *Under mixed oligopoly in quantities and homogeneous good ($\beta = 1$), the presence of even just one Coop in the market implies that all PMFs become inactive, that the output is greater than that obtained with all PMFs and that the total welfare of the economy is maximized.*

Proof. For the first result, note that, when $\beta = 1$, conditions (3.3) and (3.4) imply the following best-reply functions:

$$x_j \left(\sum_{h \in N \backslash M} x_h \right) = \frac{\alpha - (n-m)\, x_h - c}{m}, \ \forall j \in M, \tag{3.11}$$

$$x_h \left(\sum_{j \in M} x_j \right) = \frac{\alpha - m x_j - c}{(n-m+1)}, \ \forall h \in N \backslash M. \tag{3.12}$$

From which:

$$\begin{cases} x_j^* = \dfrac{\alpha - c}{m} \\ x_h^* = 0. \end{cases} \tag{3.13}$$

The total output of the economy is thus given by

$$\sum_{j \in M} x_j^* + \sum_{h \in N \backslash M} x_h^* = m \frac{\alpha - c}{m} + 0 = (\alpha - c) > \sum_k \overline{x}_k = n\,(\alpha - c)\,/\,(n+1)$$

The welfare of the economy is defined as the sum of consumers' surplus and of firms' profits, which in this case are equal to zero. Using (3.1) and (3.13) this becomes:

$$\begin{aligned} TW &= U^i\left(x_1^*, x_2^*, .., x_n^*, y^*\right) - \sum_{k=1}^n p_k x_k^* + \sum_{k=1}^n p_k x_k^* - c \sum_{k=1}^n x_k^* = \\ &= (\alpha - c) \sum_{j \in M} x_j^* - (1/2) \left(\sum_{j \in M} \left(x_j^*\right)^2 + \sum_{j \in M} \sum_{r \neq j} \left(x_j^*\right)^2 \right) + y^* = \\ &= (\alpha - c)\, m \left(\frac{\alpha - c}{m} \right) - \frac{1}{2} m \frac{(\alpha - c)^2}{m^2} - \frac{1}{2} m\,(m-1)\, \frac{(\alpha - c)^2}{m^2} + y^* = \\ &= \frac{1}{2}\,(\alpha - c)^2 + y^*. \end{aligned}$$

which is also the maximum welfare obtainable in the market. □

Moreover, some simple results can be obtained for $\beta \in [0, 1]$.

Proposition 2. *Under a mixed oligopoly in quantities, for $\beta \in [0, 1]$ the output of a Coop is always greater than the output of a PMF, that is, $x_j^* > x_h^*$ for every $j \in M$ and $h \in N \backslash M$. Moreover, for $\beta \in [0, 1]$, $x_j^* > \overline{x}_k \geq x_h^*$.*

Proof. The first result can be easily checked by direct inspection of expressions (3.9) and (3.8). The second result can be proved by noting that, for every $j \in M$ and every $k \in N$,

$$x_j^* - \overline{x}_k = \frac{(\beta(n-m-1)+2)(\alpha-c)}{\left(\beta(n+m-3)-\beta^2(n-1)+2\right)(\beta(n-1)+2)} \tag{3.14}$$

and expression (3.14) is always strictly positive for $\beta \in [0,1]$ and $n \geq 2$.

Finally, for every $h \in N \backslash M$

$$\overline{x}_k - x_h^* = \frac{(\alpha-c)}{2+\beta(n-1)} - \frac{(\alpha-c)(1-\beta)}{2+\beta(n+m)-\beta^2(n-1)-3\beta}$$

is equal to zero for $\beta = 0$, since $\overline{x}_k(\beta=0) = x_h^*(\beta=0) = (\alpha-c)/2$,

while for $\beta = 1$, $\overline{x}_k(\beta=1) = (\alpha-c)/(n+1) > x_h^*(\beta=1) = 0$. Finally, straightforward manipulations show that for $\beta \in (0,1)$,

$$\overline{x}_k - x_h^* = \frac{(\alpha-c)\,m\beta}{\left(\beta(n+m-3)+\beta^2(1-n)+2\right)(n(\beta-1)+2)} > 0$$

whenever

$$\left(\beta(n+m-3)+\beta^2(1-n)+2\right) > 0$$

which is always satisfied for $\beta \in (0,1)$. □

3.3. **Welfare Analysis: PMFs vs. Mixed Oligopoly.** The analysis of welfare under mixed oligopoly with differentiated goods requires a careful calculation of the interacting effects of the simultaneous presence of Coops and PMFs on consumer surplus and profits in all markets. By the property of quasi-linear preferences, consumers' welfare can be measured exactly through consumers' surplus and this in turn corresponds to the utility of consumers purchasing the goods. Therefore, we can proceed by computing the welfare under all various forms of oligopoly.

In a pure PMF oligopoly, for every k-th good produced, the total welfare (TW_k), calculated as the sum of consumers' surplus and firms' profits is given by

$$\begin{aligned} TW_k^{PMF} &= \int_0^{x_k^*} p_k(\tau)d\tau - p_k(\overline{x}_1, \overline{x}_2,, .., \overline{x}_n)\, x_k^* + p_k(\overline{x}_1, \overline{x}_2,, .., \overline{x}_n)\, \overline{x}_k - c\overline{x}_k \\ &= U(\overline{x}_1, \overline{x}_2,, .., \overline{x}_n) - c\overline{x}_k + \overline{y}. \end{aligned}$$

As a result, summing up the total welfare generated in all n symmetrically differentiated markets and using utility function (3.1), we obtain

$$TW^{PMF} = (\alpha-c)\sum_{k=1}^{n} \overline{x}_k - (1/2)\left[\sum_{i=1}^{n} (\overline{x}_k)^2 + \beta \sum_{k=1}^{n} \overline{x}_k \sum_{r \neq k} \overline{x}_r\right] + \overline{y},$$

which, by symmetry, can written as

$$TW^{PMF} = (\alpha - c)\, n \cdot \overline{x}_k - (1/2)\left[n\left(\overline{x}_k\right)^2 + \beta n(n-1)\overline{x}_k^2\right] + \overline{y}. \tag{3.15}$$

Under mixed oligopoly, the total welfare generated in every j-th Coop market is given by the area under the demand curve for the equilibrium quantity such that $p_j(x_1^*, x_2^*, .., x_n^*) = c_j$,

$$TW_j^{COOP} = \int_0^{x_j^*} p_j(\tau)d\tau - cx_j^*$$

which, using utility function (3.1) can simply be expressed as

$$TW_j^{COOP} = \sum_{j\in M} (\alpha - c)\cdot x_j^* - (1/2)\left[\sum_{j\in M}\left(x_j^*\right)^2 + \beta \sum_{j\in M} x_r^* \sum_{r\neq j} x_r^*\right].$$

Finally, the total welfare under mixed oligopoly can be computed as

$$\sum_{h\in N\setminus M} TW_h^{PMF} + \sum_{j\in M} TW_j^{COOP} =$$

$$= \sum_{h\in N\setminus M} (\alpha - c)\cdot x_h^* - (1/2)\left[\sum_{h\in N\setminus M} (x_h^*)^2 + \beta \sum_{h\in N\setminus M} x_h^* \sum_{r\neq h} x_r^*\right] +$$

$$+ \sum_{j\in M} (\alpha - c)\cdot x_j^* - (1/2)\left[\sum_{j\in M}\left(x_j^*\right)^2 + \beta \sum_{j\in M} x_r^* \sum_{r\neq j} x_r^*\right] + y^*,$$

which, by the symmetry of every j-th Coop and every h-th PMF, can be written as

$$TW^{MO} = (n-m)\left[(\alpha - c)\, x_h^* - (1/2)\left(x_h^{*2} + \beta m x_j^* x_h^* + \beta(n-m-1)x_h^{*2}\right)\right] +$$
$$+ m\left[(\alpha - c)\, x_j^* - (1/2)\left(x_j^{*2} + \beta\,(n-m)\, x_j^* x_h^* + \beta(m-1)x_j^{*2}\right)\right] + y^*.$$

Now, by (3.5), (3.8) and (3.9) we obtain the following expressions for the total welfare, respectively, in a pure PMFs' oligopoly

$$TW^{PMF} = (\alpha - c)\, n \tfrac{(\alpha-c)}{2+\beta(n-1)} - \frac{1}{2} n \tfrac{(\alpha-c)^2}{(2+\beta(n-1))^2} - \frac{\beta}{2} n(n-1) \tfrac{(\alpha-c)^2}{(2+\beta(n-1))^2} + \overline{y} =$$

$$= \frac{1}{2}\frac{n\,(\alpha - c)^2\,(3 + \beta\,(n-1))}{(2 + \beta\,(n-1))^2} + \overline{y}, \tag{3.16}$$

in a pure Coops' oligopoly

$$TW^{COOP} = (\alpha - c)\, \tfrac{n(\alpha-c)}{1+\beta(n-1)} - \frac{1}{2} n \tfrac{(\alpha-c)^2}{(1+\beta(n-1))^2} - \frac{\beta}{2} n(n-1) \tfrac{(\alpha-c)^2}{(1+\beta(n-1))^2} + \overline{y} =$$

$$= \frac{1}{2}\frac{n\,(\alpha - c)^2}{1 + \beta\,(n-1)} + \widetilde{y} \tag{3.17}$$

and in a mixed oligopoly with m Coops and $(n-m)$ PMFs,

$$\begin{aligned}
TW^{MO} \quad = \quad & (n-m)\left(\alpha-c\right)\tfrac{(1-\beta)(\alpha-c)}{2+(n+m-3)\beta-(n-1)\beta^2}- \\
& -\tfrac{1}{2}(n-m)\left(\tfrac{(1-\beta)(\alpha-c)}{2+(n+m-3)\beta-(n-1)\beta^2}\right)^2- \\
& -\frac{\beta}{2}(n-m)m\tfrac{(1-\beta)(2-\beta)(\alpha-c)^2}{\left(2+(n+m-3)\beta-(n-1)\beta^2\right)^2}- \\
& -\frac{\beta}{2}(n-m)(n-m-1)\left(\tfrac{(1-\beta)(\alpha-c)}{2+(n+m-3)\beta-(n-1)\beta^2}\right)^2+ \\
& +m\left(\alpha-c\right)\tfrac{(2-\beta)(\alpha-c)}{2+(n+m-3)\beta-(n-1)\beta^2}-\tfrac{1}{2}m\left(\tfrac{(2-\beta)(\alpha-c)}{2+(n+m-3)\beta-(n-1)\beta^2}\right)^2- \\
& -\frac{\beta}{2}m\left(m-1\right)\left(\tfrac{(2-\beta)(\alpha-c)}{2+(n+m-3)\beta-(n-1)\beta^2}\right)^2- \\
& -\frac{\beta}{2}m\left(n-m\right)\tfrac{(1-\beta)(2-\beta)(\alpha-c)^2}{\left(2+(n+m-3)\beta-(n-1)\beta^2\right)^2}+y^*,
\end{aligned}$$

which, in turn, can be expressed as

(3.18)

$$TW^{MO}=\underbrace{\tfrac{1}{2}\tfrac{(n-m)(\alpha-c)^2(1-\beta)\left(3+\beta(n+m-4)-\beta^2(n-1)\right)}{\left(2+\beta(n+m-3)-\beta^2(n-1)\right)^2}}_{\sum_{h\in N\setminus M}TW_h}+\underbrace{\tfrac{1}{2}\tfrac{m(\alpha-c)^2(2-\beta)}{2+\beta(n+m-3)-\beta^2(n-1)}}_{\sum_{j\in M}TW_j}+y^*.$$

Expression (3.18) illustrates the fact that in a mixed oligopoly the welfare is the sum of the welfare generated in the $(n-m)$ markets in which PMFs are active and which generated in the m markets in which retail cooperatives are, in turn, active.

For illustrative purposes in the next section we will focus on the case of mixed duopoly compared, respectively, to a pure PMF and a pure Coop duopoly. We show that the presence of Coops can be relatively more beneficial in some circumstances than in others and, in particular, for specific levels of product differentiation.

3.4. **The Duopoly Case.** The main feature of a mixed duopoly between a PMF and a Coop is which to break the symmetry of firms' behaviour. When competition is *à la* Cournot, the best-reply of the Coop is

$$x_j\left(x_h\right)=\left(\alpha-\beta x_h-c\right), \tag{3.19}$$

and which of the PMF is

$$x_h\left(x_j\right)=\frac{1}{2}\left(\alpha-\beta x_j-c\right). \tag{3.20}$$

It is clear that a PMF usually possesses a downward sloping best-reply steeper than a Coop's and therefore its equilibrium quantity is in general lower, that is

$$x_h^*=\frac{\left(1-\beta\right)\left(\alpha-c\right)}{2-\beta^2} \tag{3.21}$$

for the PMF and

$$x_j^* = \frac{(2-\beta)(\alpha - c)}{2-\beta^2} \tag{3.22}$$

for the Coop. It is important to notice that the asymmetry of the two firms' quantities is mitigated when goods are either highly differentiated (low β) or highly complement ($\beta < 0$) since

$$x_j^* - x_h^* = \frac{(\alpha - c)}{2-\beta^2}.$$

This is because high product differentiation mitigates competition and aggressive behaviour among firms, hence decreasing their equilibrium output. If we compare the output of firms under mixed duopoly to which of a *pure PMF duopoly* (denoted $\overline{x}$)

$$\overline{x}_k = \frac{(\alpha - c)}{2+\beta} \tag{3.23}$$

as well as to the case of a *pure Coop duopoly* (denoted $\widetilde{x}$),

$$\widetilde{x}_k = \frac{(2-\beta)(\alpha - c)}{2+\beta-\beta^2} \tag{3.24}$$

we note that since quantities are strategic substitutes (Bulow *et al.*, 1985) Coop's output is positively affected by the presence of a different type of competitor, while PMF's is not. In terms of price, a PMF competing with a Coop experiences a price reduction with respect to a pure PMF market, given that

$$\begin{aligned} p_h(x^*) &= \frac{\alpha + c - 2(\alpha - c)\beta + (\alpha - 2c)\beta^2}{2-\beta^2} \\ p_j(x^*) &= c \\ p_k(\overline{x}) &= \frac{\alpha + \beta(1+c)}{2+\beta}. \end{aligned}$$

Moreover, it is easy to see that

$$p_h(x^*) - p_k(\overline{x}) = \frac{(\alpha - c)\left(\beta + \beta^2 - 3\right)\beta}{\left(2-\beta^2\right)(\beta+2)} \leq 0 \text{ for } \beta \in [0,1].$$

Given the above comparative statics, we can now compare the total welfare generated in all pure and mixed cases. By (3.16), (3.17) and (3.18) we obtain that total welfare is, respectively,

$$TW^{PMF} = \frac{(\alpha - c)^2 (3+\beta)}{(2+\beta)^2}$$

in a pure PMF duopoly,

$$TW^{COOP} = \frac{(\alpha - c)^2}{(1+\beta)}$$

in a pure Coop duopoly and

$$TW^{MO} = \frac{1}{2}\frac{(\alpha - c)^2 \left(7 + \beta\left(2\beta^2 - 2\beta - 6\right)\right)}{\left(2-\beta^2\right)^2}$$

in a mixed duopoly.

The picture below shows that in term of total welfare a pure Coop duopoly (continuous line) outperforms both a pure PMF duopoly and a mixed duopoly for any degree of good differentiation. This is obvious, given that a pure Coop basically acts as a welfare maximizer. As already proved in proposition 1, under mixed duopoly (dotted line) for $\beta = 1$ (homogeneous goods) only the Coop remains in the market and the welfare is, therefore, maximized. Moreover, it can be noticed that the relative efficiency of the mixed duopoly with respect to the pure PMF duopoly (circled line) is higher when goods are either complement ($\beta < 0$) or highly differentiated. When goods become more and more homogeneous, the welfare loss determined in a pure PMF with respect to the mixed duopoly or to the pure Coop duopoly, decreases progressively, although never vanishes. Similarly, the mixed oligopoly approximates the maximum social welfare better and better for goods becoming increasingly substitute.

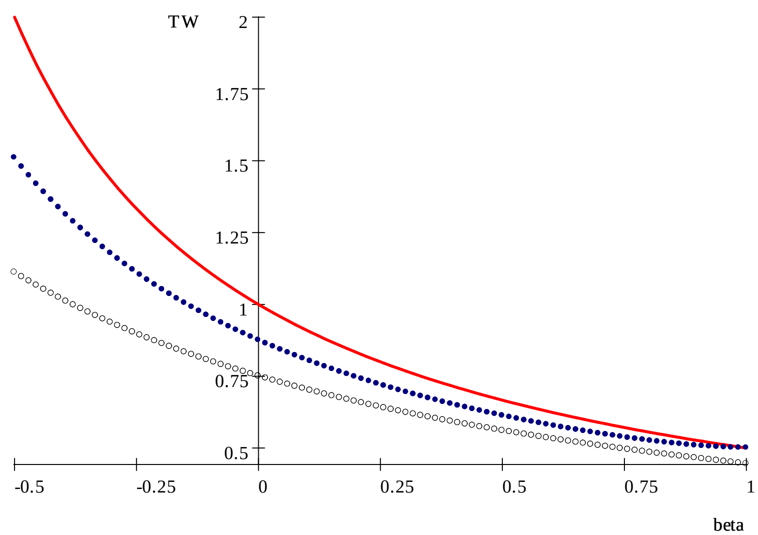

Figure 1 - Pure PMF (circled line), pure Co-op (continuous line) and mixed duopoly total welfare (dotted line) for $(\alpha - c) = 1$ and $\beta = [-0.5, 1]$.

3.5. **Welfare Comparison with More than Two Firms.** With more than two firms that compete *à la* Cournot the results obtained above continue to hold. In picture 2 we show a simulation with three firms competing in quantities. Again, a pure Coop market outperforms all other possible market forms for any degree of good differentiation. However, a simple analytical comparison shows that when goods are substitute ($\beta > 0$) in term of total welfare the pure Coop oligopoly becomes less and less advantageous with respect to a pure PMF oligopoly when both n and β increase. When competition is high (which happens for high n and β) the different forms of market do not perform too differently, and the welfare generated is not too dissimilar. This is expressed in the next proposition.

Proposition 3. *When the number of firms in the market increases and goods become increasingly substitutes (higher β), the difference between welfare generated in a pure Coop oligopoly and which generated in a pure PMF oligopoly, decreases progressively.*

Proof. Straightforward manipulations show that the following expression

$$TW^{COOPs} - TW^{PMFs} = \frac{1}{2}\frac{n(\alpha-c)^2}{1+\beta(n-1)} - \frac{1}{2}\frac{n(\alpha-c)^2(3+\beta(n-1))}{(2+\beta(n-1))^2} =$$

$$= \frac{1}{2}\frac{n\,(\alpha-c)^2}{(1+\beta\,(n-1))\,(2+\beta\,(n-1))^2} \tag{3.25}$$

is monotonically decreasing in both β and n for $n > 1$ and $1 \geq \beta > 0$. □

When goods are complements, ($\beta \leq 0$), the effect of a higher number of firms on (3.25) becomes positive.

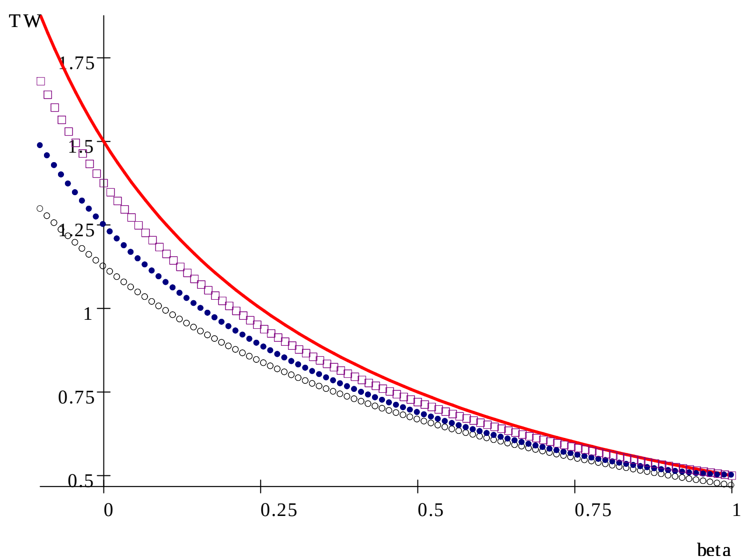

Figure 2 - Pure PMF (ircled line) pure Co-op (continuous rline) and mixed triopoly total welfare with $m = 1$ (dotted line), $m = 2$ (squared line) for $n = 3$, $(\alpha - c) = 1$ and $\beta = [-0.1, 1]$.

As shown in picture 2, with more than two firms the presence of Coops are still highly beneficial in terms of social welfare. It can also be noticed that when both the number of firms and the substitution rate among goods increase, the relative advantage of Coops in terms of welfare shrinks progressively. Therefore, if the model is not too far from reality and consumer cooperatives arise with the purpose to meet consumers' needs, we should expect them establishing their business mostly in highly monopolistic markets in which goods are either highly differentiated or complements. In the next section, we will consider the case of price competition.

4. Competition in Prices

It can be interesting to compare the case of quantity competition to that of price competition in order to see if some differences arise. One obvious relevant difference is the fact that, when goods are perfectly homogeneous, Bertrand competition yields the extreme prediction of marginal pricing regardless of the objective functions of firms competing in the market.

4.1. **Oligopoly with all PMFs.** When all firms are PMFs, we firstly obtain the direct demand for each k-th good as function of prices,

$$x_k\left(p_1, p_2, .., p_n\right) = \frac{\alpha\left(1-\beta\right) - p_k - (n-2)\beta p_k + \beta\sum_{h\neq k} p_h}{\left(1-\beta\right)\left((n-1)\beta+1\right)}$$

for $k = 1, 2, .., n$ and $\beta \neq 1$. As a result, every PMF's profit function can be written as

$$\pi_k\left(p_1, ., p_n\right) = \left(p_k - c\right)\left(\frac{\alpha(1-\beta)-p_k-(n-2)\beta p_k+\beta\sum_{h\neq k} p_h}{(1-\beta)((n-1)\beta+1)}\right). \tag{4.1}$$

Differentiating (4.1) with respect to p_k, yields the best-reply of every k-th PMF as

$$p_k(p_{-k}) = \frac{1}{2}\frac{\alpha\left(1-\beta\right) + c\left(n-2\right)\beta + c + \beta p_{-k}}{\beta\left(n-2\right)+1}$$

where $p_{-k} = (p_1, p_2, .., p_{k-1}, p_{k+1}, .., p_n)$.

By symmetry, the Nash equilibrium price of every k-th PMF is obtained as

$$\overline{p}_k = \frac{\left(\alpha\left(1-\beta\right) + \beta c\left(n-2\right) + c\right)}{\beta\left(n-3\right)+2} \tag{4.2}$$

with associated quantities:

$$x_k\left(\overline{p}_1, \overline{p}_2, .., \overline{p}_n\right) = \frac{\left(\alpha - c\right)\left(1+\beta\left(n-2\right)\right)}{\left(1+\beta\left(n-1\right)\right)\left(2+\beta\left(n-3\right)\right)} \tag{4.3}$$

and profits

$$\pi_k\left(\overline{p}_1, ., \overline{p}_n\right) = \left(\overline{p}_k - c\right)\cdot x_k\left(\overline{p}_1, ., \overline{p}_n\right) = \frac{\left(\alpha-c\right)^2\left(1-\beta\right)\left(1+\beta\left(n-2\right)\right)}{\left(2+\beta\left(n-3\right)\right)^2\left(1+\beta(n-1)\right)}.$$

4.2. **Mixed Oligopoly with Price Competition.** Again we imagine that $m \leq n$ firms in the market behave as Coops. Using (3.1) and (3.4), we obtain the following direct demands for every PMF $h \in N \backslash M$,

$$x_h(p_1, ., p_n) = \frac{\alpha(1-\beta) - p_h - \beta(n-2)p_h + \beta \sum_{k \in \{N \backslash M\} \backslash h} p_k + m\beta c}{(1-\beta)(1+\beta(n-1))} \tag{4.4}$$

and every Coop $j \in M$

$$x_j(p_1, ., p_n) = \frac{\alpha(1-\beta) - c - (n-m-1)\beta c + \beta \sum_{h \in N \backslash M} p_h}{(1-\beta)(1+\beta(n-1))} \tag{4.5}$$

for $\beta \neq 1$. By (4.4) we can write the profit-function of every PMF as function of prices,

$$\pi_h(p_1, ., p_n) = (p_h - c) \frac{\left(\alpha(1-\beta) - p_h - (n-2)\beta p_h + \beta \sum_{k \in \{N \backslash M\} \backslash h} p_k + m\beta c\right)}{(1-\beta)(1+\beta(n-1))}$$

and, through simple manipulations, the following mixed oligopoly equilibrium prices are obtained

$$\begin{cases} p_h^* = \dfrac{\alpha(1-\beta) + c(1+\beta(n+m-2))}{2+\beta(n+m-3)} \\ \\ p_j^* = c, \end{cases} \tag{4.6}$$

with associated quantities, respectively, for PMFs

$$x_h(p_1^*, p_2^*, .., p_n^*) = \frac{(\alpha - c)(1+\beta(n-2))}{(1+\beta(n-1))(2+\beta(n+m-3))} \tag{4.7}$$

and Coops

$$x_j(p_1^*, p_2^*, .., p_n^*) = \frac{(\alpha - c)(2+\beta(2n-3))}{(1+\beta(n-1))(2+\beta(n+m-3))}, \tag{4.8}$$

with every $h \in N \backslash M$ equilibrium profits given by

$$\pi_h(p_1^*, p_2^*, .., p_n^*) = \frac{(\alpha - c)^2(1-\beta)(\beta(n-2)+1)}{(2+\beta(n+m-3))^2(1+n\beta-\beta)}.$$

A few comparisons can now be made.

Proposition 4. *Under price competition and $\beta \in [0,1]$, the mixed oligopoly prices are for all firms either lower or equal than pure PMF oligopoly prices, that is, $\overline{p}_k \geq p_h^* \geq p_j^*$ for every $j \in M$, $h \in N \backslash M$ and $k = 1, 2, ..n$. Moreover, $x_j(p^*) \geq x_k(\overline{p}) \geq x_h(p^*)$.*

Proof. By expressions (4.2), (4.6) and by the property of Bertrand equilibrium, when goods are homogeneous ($\beta = 1$) there is no difference between mixed and pure oligopoly equilibrium prices, since $\overline{p}_k = p_j^* = p_h^* = c$. When goods are independent ($\beta = 0$) all PMFs behave as monopolists under both pure and mixed oligopoly, with $p_h^* = \overline{p}_k = \dfrac{a+c}{2}$ while, also in this case, Coops behave as perfectly competitive firms, setting $p_j^* = c$. Moreover, for $\beta \in (0, 1)$ it is easy to check that

$$(\overline{p}_k - p_h^*) = \frac{(\alpha - c)(1-\beta) m\beta}{(2+\beta(n+m-3))(2+\beta(n-3))},$$

which is zero for $m = 0$ and is monotonically increasing in the number of Coops, since

$$\frac{d(\overline{p}_k - p_h^*)}{dm} = \frac{(1-\beta)(\alpha - c)\beta}{(2+\beta(n+m-3))^2} > 0$$

for $n \geq 1$. About the second group of results, note that, for $\beta = 0$

$$x_k(\overline{p}) = x_h(p^*) = \frac{1}{2}(\alpha - c)$$

and, for every j-th Coops,

$$x_j(p^*, \beta = 0) = (\alpha - c), \tag{4.9}$$

and therefore

$$x_j(p^*, \beta = 0) > x_h(p^*, \beta = 0) = x_k(\overline{p}, \beta = 0).$$

Moreover, for $\beta = 1$, in term of output all types of oligopoly perform similarly with

$$x_k(\overline{p}, \beta = 1) = x_h(p^*, \beta = 1) = x_j(p^*, \beta = 1) = \frac{(\alpha - c)}{n}.$$

When $\beta \in (0, 1)$, a simple inspection of (4.3) and (4.7) shows that, for $m \geq 1$,

$$x_k(\overline{p}) > x_h(p^*).$$

Finally, for $\beta \in (0, 1)$ we have that

$$x_j(p^*) - x_k(\overline{p}) = \frac{\left(\beta(3n-m-5) + \beta^2\left(2m-4n+3-mn+n^2\right)+2\right)(\alpha - c)}{(2+\beta(n+m-3))(1+\beta(n-1))(2+\beta(n-3))}$$

whose both numerator and denominator are strictly positive within the defined range of parameters. □

4.3. **Welfare Comparison under Price Competition.** For sake of brevity we relegate in appendix all calculations of total welfare with price competition under both pure and mixed oligopoly. We report here the results, which are not too dissimilar from those obtained for the case of quantity competition. Total welfare under mixed oligopoly with an arbitrary number of PMFs and Coops competing in prices is obtained as

(4.10)
$$TW^{MO} = \underbrace{\tfrac{1}{2}\tfrac{(n-m)(c-\alpha)^2(3+\beta(n+m-4))(1+\beta(n-2))}{(2+\beta(n+m-3))^2(1+\beta(n-1))}}_{\sum_{h\in N\setminus M}TW_h} + \underbrace{\tfrac{1}{2}\tfrac{m(\alpha-c)^2(2+\beta(2n-3))}{(1+\beta(n-1))(2+\beta(n+m-3))}}_{\sum_{j\in M}TW_j}$$

where in expression above we have again decomposed the total welfare in two distinct parts. Setting in (4.10) $m=0$ we can obtain the pure PMF oligopoly welfare under price competition as

$$TW^{PMF} = \frac{1}{2}\frac{n\,(\alpha-c)^2\,(1+\beta\,(n-2))\,(3+\beta\,(n-4))}{(1+\beta\,(n-1))\,(2+\beta\,(n-3))^2}$$

while, setting $n=m$ we obtain the pure Coop welfare as

$$TW^{COOP} = \frac{1}{2}\frac{(\alpha-c)^2\,n}{(1+\beta\,(n-1))^2}.$$

It can be noticed that the pure Coop oligopoly always yields the economy maximum welfare no matter if competition is *à la* Cournot or *à la* Bertrand. Again, for illustrative purposes, in the following section we present the duopoly case in order to underline the main differences between price and quantity competition.

4.4. **The Duopoly Case with Price Competition.** When both firms in the market are PMFs, we can compute the following direct demand for every k-th good ($k=1,2$) as function of the price vector (p_1,p_2), for $\beta\neq 1$

$$x_k\,(p_1,p_2) = \frac{\alpha\,(1-\beta)-p_1+\beta p_2}{\left(1-\beta^2\right)}$$

and profit functions

$$\begin{aligned}\pi_1\,(p_1,p_2) &= (p_1-c)\left(\frac{\alpha\,(1-\beta)-p_1+\beta p_2}{\left(1-\beta^2\right)}\right)\\ \pi_2\,(p_1,p_2) &= (p_2-c)\left(\frac{\alpha\,(1-\beta)-p_2+\beta p_1}{\left(1-\beta^2\right)}\right).\end{aligned}$$

By profit functions differentiation, we obtain the following best-replies for both firms:

$$\begin{aligned}p_1\,(p_2) &= \frac{1}{2}\frac{\alpha\,(1-\beta)+\beta p_2+c}{\left(1-\beta^2\right)}\\ p_2\,(p_1) &= \frac{1}{2}\frac{\alpha\,(1-\beta)+\beta p_1+c}{\left(1-\beta^2\right)}\end{aligned}$$

whose solution are, for $\beta \neq 1$:

$$\begin{cases} \overline{p}_1 = \overline{p}_2 = \dfrac{\alpha(1-\beta)+c}{2-\beta(1+2\beta)} \text{ for } \beta \neq 1 \\ \text{and } \overline{p}_1 = \overline{p}_2 = c \;\text{ for } \beta = 1, \end{cases}$$

with associated quantities equal to:

$$x_1(\overline{p}_1, \overline{p}_2) = x_1(\overline{p}_1, \overline{p}_2) = \frac{\alpha - c}{(1+\beta)(2-\beta)}.$$

Let us now immagine that firm 1 is a pure Coop and firm 2 a pure PMF. When the Coop distributes entirely its profits, the FOC of the representative consumer's utility becomes, respectively for good 1 and good 2:

$$\begin{aligned} \alpha - x_1 - \beta x_2 - c &= 0 \\ \alpha - x_2 - \beta x_1 - p_2 &= 0 \end{aligned}$$

from which the following demand functions are derived as function of prices, for $\beta \neq 1$:

$$\begin{aligned} x_1(p_1, p_2) &= \frac{\alpha(1-\beta) + \beta p_2 - c}{(1-\beta^2)} \\ x_2(p_1, p_2) &= \frac{\alpha(1-\beta) - p_2 + \beta c}{(1-\beta^2)}. \end{aligned}$$

Profit functions can be defined as:

$$\begin{aligned} \pi_1(p_1, p_2) &= (p_1 - c)\left(\frac{\alpha(1-\beta) + \beta p_2 - c}{(1-\beta^2)}\right) \\ \pi_2(p_1, p_2) &= (p_2 - c)\left(\frac{\alpha(1-\beta) - p_2 + \beta c}{(1-\beta^2)}\right). \end{aligned}$$

Differentiating the two expressions, the following best-reply functions are obtained, which are also the solutions of the problem:

$$p_1^* = c$$

and

$$\begin{cases} p_2^* = \frac{1}{2}(\alpha(1-\beta) + c(1+\beta)) \;\text{ for } \beta \neq 1 \\ p_2^* = c \;\text{ for } \beta = 1. \end{cases}$$

with associated quantities for $\beta \neq 1$,[8]

[8]Note that the equilibrium quanties are obtained plugging into direct demands (4.4) and (4.5) the equilibrium prices. Since the latter are not defined for $\beta = 1$, the equilibrium outputs are similarly not defined for this value. Ouput levels for $\beta = 1$ are simply defined as the firms' direct demnds obtained when all prices are set at marginal cost, i.e., $x_1(p_1^*, p_2^*) = x_2(p_1^*, p_2^*) = \frac{(\alpha - c)}{2}$.

$$
\begin{aligned}
x_1\left(p_1^*, p_2^*\right) &= \frac{1}{2}\frac{(\alpha - c)(2+\beta)}{1+\beta} \\
x_2\left(p_1^*, p_2^*\right) &= \frac{1}{2}\frac{\alpha - c}{1+\beta}.
\end{aligned}
$$

and

$$
x_1\left(p_1^*, p_2^*\right) = x_2\left(p_1^*, p_2^*\right) = \frac{(\alpha - c)}{2}
$$

for $\beta = 1$. Finally, if both firms are Coops, their equilibrium quantity will be equal to

$$
\widetilde{x}_1\left(p_1, p_2\right) = \widetilde{x}_2\left(p_1, p_2\right) = \frac{(\alpha - c)}{(\beta + 1)}.
$$

It can be noticed that, differently from the case of quantity competition, here the Coop's best-reply is completely independent of the price of its rival. Therefore, under Bertrand competition the specific leader-follower configuration in which the PMF is leader and the Coop is follower would not yield any difference when compared to the simultaneous case presented above. The same cannot be said under Cournot competition.

If we now compute the welfare yielded under duopoly, the following expressions, for $m = 0, 1, 2$, respectively, are obtained as

$$
\begin{aligned}
TW^{PMF} &= \frac{(\alpha - c)^2 (3 - 2\beta)}{(1+\beta)(2-\beta)^2}, \\
TW^{MO} &= \frac{1}{8}\frac{(\alpha - c)^2 (7+\beta)}{(1+\beta)}, \\
TW^{COOP} &= \frac{(\alpha - c)^2}{(1+\beta)}.
\end{aligned}
$$

By plotting the three expressions within the range of β does not yield particular differences with respect to the case of Cournot competition, except that here all types of markets, included the pure PMF-duopoly, yields the marginal cost pricing and then the maximum welfare for $\beta = 1$. Therefore, under Bertrand competition and homogeneous goods we observe a perfect "isomorphism" in the behaviour of all firms.

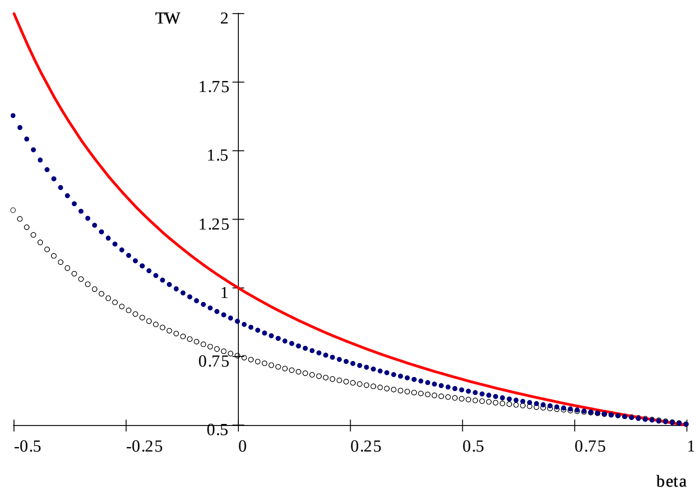

Figure 3- Pure PMF (circled line), pure Co-op (continuous line) and mixed duopoly total welfare (dotted line) for $(\alpha - c) = 1$ and $\beta = [-0.5, 1]$.

Extending the welfare comparisons to more than two firms shows that, again, the analysis is qualitatively similar to that of Cournot competition, as shown in figure 4.

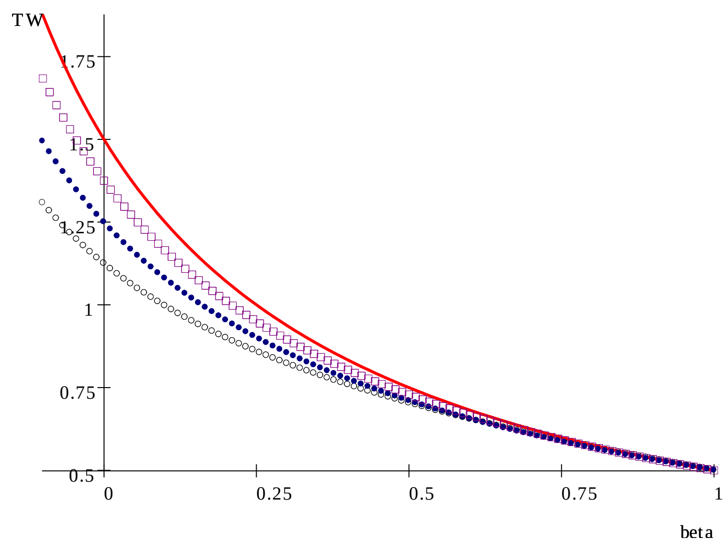

Figure 4 - Total welfare of a pure PMF (circled line), of a mixed triopoly with $m = 1$ (dotted line) and $m = 2$ (squared line) and of a pure Co-op (continuous line) for $n = 3$, $(a - c) = 1$ and $\beta = [-0.1, 1]$.

An important difference between Bertrand and Cournot competition emerges in terms of the welfare loss of a pure PMF oligopoly with respect to a pure Coop oligopoly. As shown below in figure 5, this loss is definitively larger under quantity than under price competition and the difference is particularly high when goods are reasonably homogeneous, therefore making under these circumstances the presence of at least one Coop in the market definitively more beneficial under Cournot than under Bertrand competition. Additional welfare comparisons between Cournot and Bertrand oligopolies are relegated in the appendix.

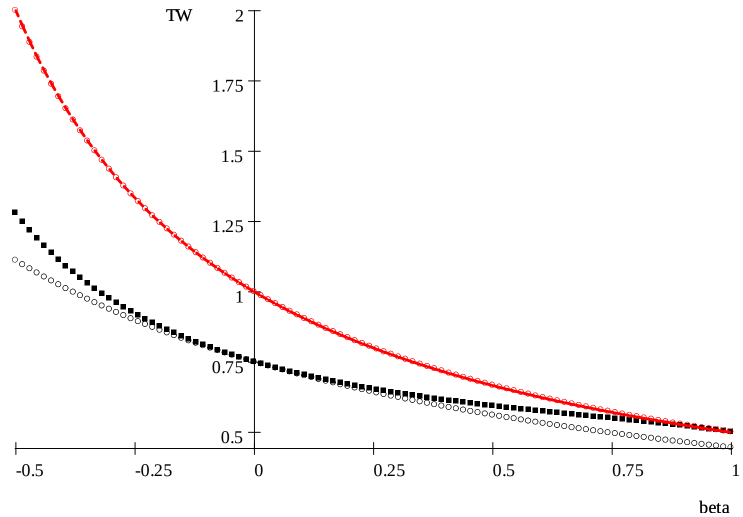

Fig.5 - Total welfare in a pure PMF duopoly market under Cournot (circled line) and Bertrand (dotted line) competion compared to a pure Co-op market (continuous line) for $(a-c)=1$ and $\beta \in [-0.5, 1]$.

5. Concluding Remarks

Although in general consumer cooperatives are well established in many countries, their actual behaviour is still largely unknown and requires additional research, in particular to understand the effects of their strategic interaction with traditional profit maximizing firms in oligopolistic markets. This paper has attempted to make a first step in this direction, showing the main effects arising in a mixed oligopoly with profit-maximizing firms and consumer cooperarives competing either *à la* Cournot or *à la* Bertand in markets with heterogeneous goods. We have shown that the presence of Coops is beneficial for industries output and social welfare in mainly two cases. The first under Cournot comeptition, when goods are perfectly homogeneous and therefore Coops behave so expansively to expel PMFs from the market, or, if interpreted differently, to oblige them to behave as perfectly competitive firms, setting a price equal to the marginal cost and making zero profit as a result. The second case is when goods are either complements or highly differentiated, for which the presence of Coops appears particularly valuable for markets, by increasing considerably their output and welfare. In this paper we have also shown that Coops affects relatively more the total welfare under Cournot than under Bertrand competition. Therefore, according to our model, consumer cooperatives should behave in a not too dissimilar way to traditional profit maximizing firms in all retail markets in which goods are highly (but not completely) homogeneous and in which competition happens mostly in prices. As a reaction to these market forces, Coops could attempt to propose their customers genuinely differentiated goods and, in this way, enhance consumers' welfare and accomplish their authentic objective function.

References

[1] Anderson R.K., S. C. Maurice, P. K. Porter (1979), "The Economics of Consumer-Managed Firms", *Southern Economic Journal*, July, 119-130.

[2] Anderson R.K., S. C. Maurice, P. K. Porter (1980), "Factor Usage by Consumer-Managed Firms", *Southern Economic Journal,* 522-528.

[3] Bulow, J., J. Geneakoplos, and P. Klemperer (1985), "Multimarket oligopoly: Strategic substitutes and complements". *Journal of Public Economics*, 93, 488–511.

[4] De Fraja, G. and F. Delbono (1990). Game Theoretic Models of Mixed Oligopoly. *Journal of Economic Surveys* 4, 1–17.

[5] Dixit, A. (1979), " A Model of Duopoly Suggesting a Theory of Entry Barriers", *Bell Journal of Economics*, 14, 383-394.

[6] Enke, S. (1945), "Consumer Cooperatives and Economic Efficiency", *American Economic Review*, March, 148-155.

[7] Fulton, M. (1989), "Cooperatives in Oligopolistic Industries: The Western Canadian Fertilizer Industry", *Journal of Agricultural Cooperation*, 4, 1-19.

[8] Fulton, M. and K. Giannakas (2001), "Organizational Commitment in a Mixed Oligopoly: Agricultural Cooperatives and Investor-Owned Firms", *American Journal of Agricultural Economics*, 83, 5, 1258-1265.

[9] Goering, G. E. (2008) "Welfare impacts of a non-profit firm in mixed commercial markets", *Economic Systems*, 32, 326–334

[10] Ireland N. and P. J. Law, (1983), "A Cournot-Nash Model of the Consumer Cooperative", *Southern Economic Journal,* 49, 706-716.

[11] Farrell, J. (1985), "Owner-Consumers and Efficiency", *Economics Letters*, 19, 303-306.

[12] Hart, O. and J. Moore (1996), "The Governance of Exchanges: Members' Cooperatives versus Outside Ownership," *Oxford Review of Economic Policy*, 12, 4, 53-69.

[13] Hart, O. and J. Moore, (1998), "Cooperatives vs. Outside Ownership", *Harvard Institute of Economic Research Working Papers* 1816, Harvard, Institute of Economic Research.

[14] Mikami, K. (2003) "Market power and the form of enterprise: capitalist firms, worker-owned firms and consumer cooperatives", *Journal of Economic Behaviour and Organization*, 52, 4, December, 533-552.

[15] Mikami, K. (2010) " Capital Procurement of a Consumer Cooperative: Role of the Membership Market", *Economic Systems*, 34, 2, 178-197.

[16] Rhodes, V. J. (1983), "The Large Agricultural Cooperative as a Competitor." *American Journal of Agricultural Economics*, 65, 1090-95.

[17] Sandler, T. and J. Tschirhart (1981), "On the number and membership size of Consumer Managed Firms", *Southern Economic Journal*, 1086-1091.

[18] Sexton, R. (1983), "Economic Considerations in Forming Cooperatives and Establishing Pricing and Financing Policies", *the Journal of Consumer Affairs*, 17, 2, 290-314.

[19] Sexton, R. I. and A. Sexton (1987), "Cooperatives as Entrants", *Rand Journal of Economics*, 18, 581-95.

[20] Sexton, R. J. (1990), "Imperfect Competition in Agricultural Markets and the Role of Cooperatives: a Spatial Analysis", *American Journal of Agricultural Economics*, 72, 709-20.

[21] Shubik, M. and R. Levitan (1971), *Market Structure and Behaviour*. Cambridge, Harvard University Press.

[22] Tennbakk, B. (1992) "The Case of Cooperative Mixed Duopoly", Institute ofor Research on labor and employment, UC Berkeley, *Working Paper Series*, 06-01-1992.

[23] Yamey, B.S. (1950) "The Price Policy of Cooperative Societies", *Economica*, February, 23-42.

[24] Vives, X. (1984), "Duopoly Information: Cournot and Bertrand", *Journal of Economic Theory*, 34, 71-94.

6. Appendix

6.1. **Total Welfare under Price Competition.** Using (3.15) and (4.3) we obtain the pure PMF-oligopoly welfare under price competition as

$$TW^{PMF} = (\alpha - c)\, n \cdot x_k(\overline{p}) - \tfrac{1}{2}\left[n\left(x_k(\overline{p})\right)^2 + \beta n(n-1) x_k^2(\overline{p})\right], \tag{6.1}$$

that is

$$\begin{aligned} TW^{PMF} &= (\alpha - c)\, n \tfrac{(\alpha-c)(1+\beta(n-2))}{(1+\beta(n-1))(2+\beta(n-3))} - \\ &\quad -\tfrac{1}{2} n \left(\tfrac{(\alpha-c)(1+\beta(n-2))}{(1+\beta(n-1))(2+\beta(n-3))}\right)^2 - \\ &\quad -\tfrac{\beta}{2} n(n-1)\left(\tfrac{(\alpha-c)(1+\beta(n-2))}{(1+\beta(n-1))(2+\beta(n-3))}\right)^2 \end{aligned}$$

which, after some manipulations yields

$$TW^{PMF} = \tfrac{1}{2}\tfrac{n(\alpha-c)^2(1+\beta(n-2))(3+\beta(n-4))}{(1+\beta(n-1))(2+\beta(n-3))^2}.$$

In a pure Coops' Bertrand oligopoly with n firms we obtain

$$\begin{aligned} TW^{COOP} &= n(\alpha - c)\tfrac{(\alpha-c)}{(1+\beta(n-1))} - \tfrac{1}{2} n \left(\tfrac{(\alpha-c)}{(1+\beta(n-1))}\right)^2 - \\ -\tfrac{\beta}{2} n(n-1)\left(\tfrac{(\alpha-c)}{(1+\beta(n-1))}\right)^2 &= \tfrac{1}{2}\tfrac{n(\alpha-c)^2}{(1+\beta(n-1))} \end{aligned}$$

Moreover, using (4.7) and (4.8) and knowing that

$$\begin{aligned} &TW^{MO} = (n-m)\left[(\alpha-c)\, x_h^* - \tfrac{1}{2}\left(x_h^{*2} + \beta m x_j^* x_h^* + \beta(n-m-1) x_h^{*2}\right)\right] + \\ &+m\left[(\alpha-c)\, x_j^* - \tfrac{1}{2}\left(x_j^{*2} + \beta(n-m)\, x_j^* x_h^* + \beta(m-1) x_j^{*2}\right)\right]. \end{aligned}$$

we obtain

$$\begin{aligned} TW^{MO} &= (n-m)(\alpha-c)\tfrac{(\alpha-c)(1+\beta(n-2))}{(1+\beta(n-1))(2+\beta(n+m-3))} - \tfrac{1}{2}(n-m) \\ &\quad \left(\tfrac{(\alpha-c)(1+\beta(n-2))}{(1+\beta(n-1))(2+\beta(n+m-3))}\right)^2 - \\ &\quad -\tfrac{\beta}{2}(n-m)m\tfrac{(\alpha-c)^2(1+\beta(n-2))(2+\beta(2n-3))}{(1+\beta(n-1))(2+\beta(n+m-3))^2} - \\ &\quad -\tfrac{\beta}{2}(n-m)(n-m-1) \\ &\quad \left(\tfrac{(\alpha-c)(1+\beta(n-2))}{(1+\beta(n-1))(2+\beta(n+m-3))}\right)^2 + \\ &\quad +(\alpha-c)\, m \tfrac{(\alpha-c)(2+\beta(2n-3))}{(1+\beta(n-1))(2+\beta(n+m-3))} - \\ &\quad -\tfrac{1}{2} m \left(\tfrac{(\alpha-c)(2+\beta(2n-3))}{(1+\beta(n-1))(2+\beta(n+m-3))}\right)^2 - \\ &\quad -\tfrac{\beta}{2}(n-m)\, m \tfrac{(\alpha-c)^2(1+\beta(n-2))(2+\beta(2n-3))}{(1+\beta(n-1))(2+\beta(n+m-3))^2} + \\ &\quad +\tfrac{\beta}{2} m(m-1)\left(\tfrac{(\alpha-c)(2+\beta(2n-3))}{(1+\beta(n-1))(2+\beta(n+m-3))}\right)^2 \end{aligned}$$

which can be decomposed in the total welfare obtained in all markets in which PMFs operate

$$\begin{aligned}\sum_{h\in N\setminus M} TW_h &= (n-m)\,(\alpha-c)\,\tfrac{(\alpha-c)(\beta(n-2)+1)}{(\beta(n-1)+1)(\beta(n+m-3)+2)} - \\ &\quad -\tfrac{1}{2}(n-m)\left(\tfrac{(\alpha-c)(\beta(n-2)+1)}{(\beta(n-1)+1)(\beta(n+m-3)+2)}\right)^2 \\ &\quad -\tfrac{\beta}{2}(n-m)m - \\ &\quad -\tfrac{\beta}{2}(n-m)(n-m-1)\left(\tfrac{(\alpha-c)(\beta(n-2)+1)}{(\beta(n-1)+1)(\beta(n+m-3)+2)}\right)^2\end{aligned}$$

i.e.,

$$\sum_{h\in N\setminus M} TW_h = \tfrac{1}{2}\tfrac{(n-m)(c-\alpha)^2(3+\beta(n+m-4))(1+\beta(n-2))}{(2+\beta(n+m-3))^2(1+\beta(n-1))},$$

and the total welfare obtained in all markets in which Coops operate,

$$\begin{aligned}\sum_{j\in M} TW_j &= m\,(\alpha-c)\,\tfrac{(\alpha-c)(2+\beta(2n-3))}{(\beta(n-1)+1)(\beta(n+m-3)+2)} - \\ &\quad -\tfrac{1}{2}m\left(\tfrac{(\alpha-c)(2+\beta(2n-3))}{(\beta(n-1)+1)(\beta(n+m-3)+2)}\right)^2 \\ &\quad -\tfrac{\beta}{2}m\,(m-1)\left(\tfrac{(\alpha-c)(2+\beta(2n-3))}{(\beta(n-1)+1)(\beta(n+m-3)+2)}\right)^2 \\ &\quad -\tfrac{\beta}{2}m\,(n-m)\,\tfrac{(\alpha-c)^2(1+\beta(n-2))(2+\beta(2n-3))}{((1+\beta(n-1))(2+\beta(n+m-3)))^2},\end{aligned}$$

i.e.,

$$\sum_{j\in M} TW_j = \tfrac{1}{2}\tfrac{m(\alpha-c)^2(2+\beta(2n-3))}{(1+\beta(n-1))(2+\beta(n+m-3))}.$$

Therefore, we can write

(6.2)

$$TW^{MO} = \underbrace{\tfrac{1}{2}\tfrac{(n-m)(a-c)^2(3+\beta(n+m-4))(1+\beta(n-2))}{(2+\beta(n+m-3))^2(1+\beta(n-1))}}_{\sum_{h\in N\setminus M} TW_h} + \underbrace{\tfrac{1}{2}\tfrac{m(\alpha-c)^2(2+\beta(2n-3))}{(1+\beta(n-1))(2+\beta(n+m-3))}}_{\sum_{j\in M} TW_j}.$$

A welfare comparison between Bertrand (6.2) and Cournot total welfare (3.18) for $(a-c)=1$, yields the following expression:

$$TW_p^{MO} - TW_q^{MO} = \tfrac{1}{2}\tfrac{\left(6\beta-2m\beta-2n\beta-\beta^2+n\beta^2-4\right)(n-m)(n-1)(1+\beta(m-1))(\beta-1)\beta^2}{(2+\beta(n+m-3))^2\left(m\beta-3\beta+n\beta+\beta^2-n\beta^2+2\right)^2(1+\beta(n-1))}.$$

Taking for simplicity the duopoly case ($n=2$) with both $m=0$ (pure PMF duopoly)

$$TW_p^{PMF} - TW_q^{PMF} = \tfrac{\left(4-\beta^2-2\beta\right)\beta^2}{(\beta+2)^2(\beta-2)^2(\beta+1)},$$

and $m=1$ (mixed duopoly), we obtain:

$$TW_p^{MO} - TW_q^{MO} = \tfrac{1}{8}\tfrac{(\beta+2)(\beta-2)(\beta-1)\beta^2}{\left(\beta^2-2\right)^2(\beta+1)}.$$

We can plot the two expressions above for $\beta = [-0.5, 1]$.

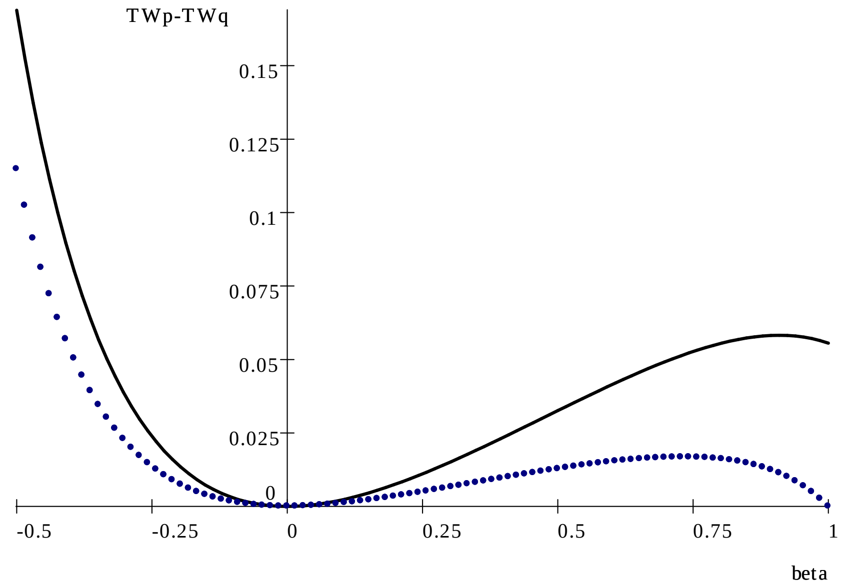

Fig. 6 - Total Welfare difference between Bertrand and Cournot comeptition under both pure PMF duopoly (continuous line) and mixed duopoly (dotted line) for $(a - c) = 1$, $n = 2$ and $\beta = [-0.5, 1]$.

The first thing to notice is that both expressions are not monotonic in β. Moreover, the welfare differences between price and quantity competition are in general larger under pure PMF duopoly than under mixed duopoly. In both cases such a difference is high when goods are complements. When goods are substitutes, we have that in a pure PMF duopoly the difference in welfare between Bertand and Cournot competition increases with beta. Conversely, In a mixed duopoly such a difference first increases and then decreases to finally disappear for $\beta = 1$. Note that for $n > 2$, the qualitative results shown above continue to hold.